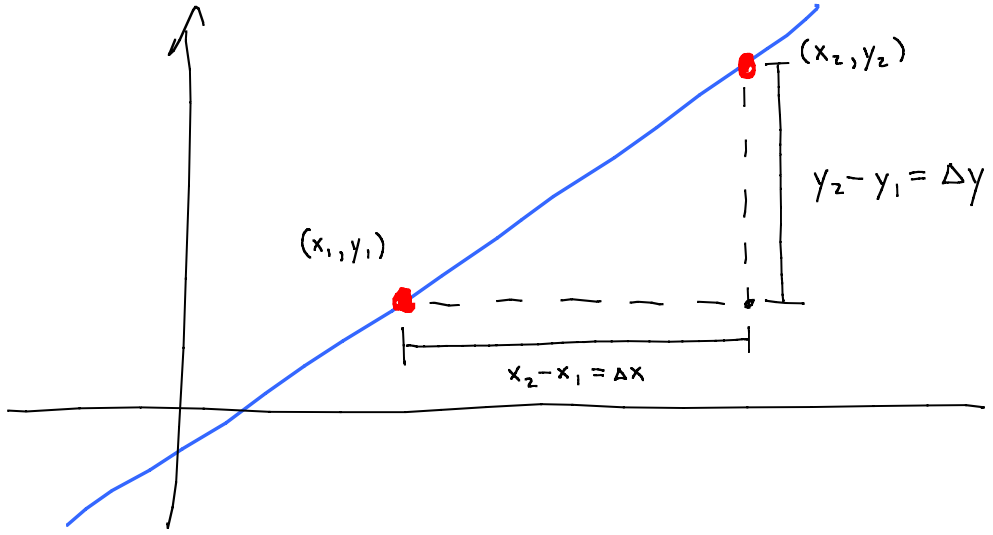
Calc 1 Lecture 2 - Zoo of Functions

Note Title 9/8/2008

Linear Functions: A function whose graph is a line.

Slope of a line:

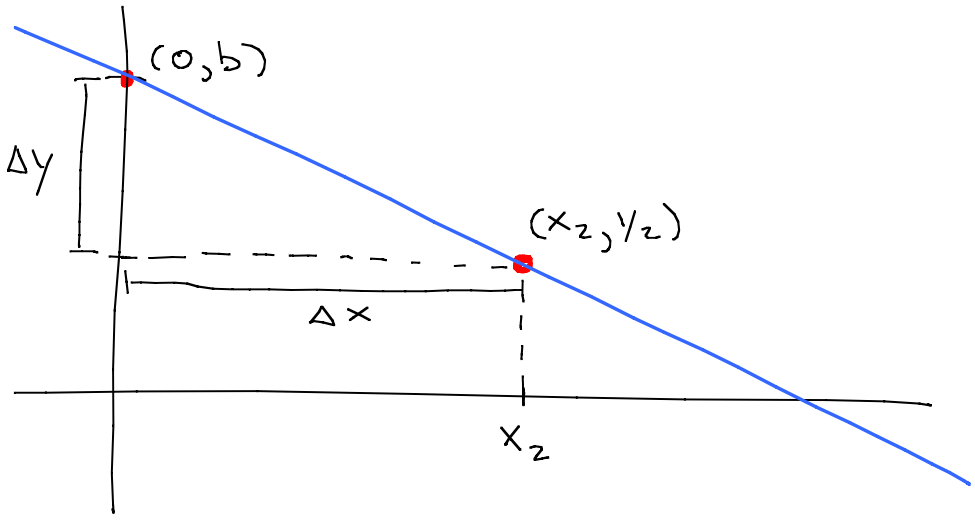


The slope of the line is $\Delta y / \Delta x$
sometimes called "Rise over Run"

For a line, the choice of points does not matter - you get the same slope.

$\Delta y / \Delta x$ is the change in y over the change in x
So when we change x by a fixed amount, we always get the same change in y-values, no matter which x-value we start with.

In particular:



so $\frac{\Delta y}{\Delta x} = \frac{y_2 - b}{x_2 - 0}$

$(x_2 - 0)\ \frac{\Delta y}{\Delta x} = y_2 - b$

$y_2 = \frac{\Delta y}{\Delta x} x_2 + b$ ← true for any value of x_2

so: $y = mx + b$

m = slope = $\frac{\Delta y}{\Delta x}$

Slope-intercept form of a line

Also: $y = m(x - x_1) + y_1$ Point-slope form for point (x_1, y_1) on the line

Increasing line: $m > 0$

Decreasing line: $m < 0$

Important example: $D = rt$

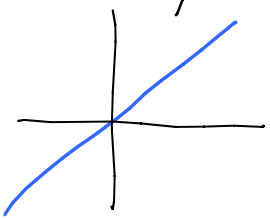
distance equals rate times time

When the rate is fixed, distance is a linear function of time.

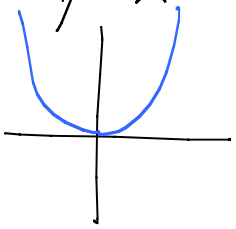
Power functions: $y = K x^{\alpha}$

K and α both real numbers

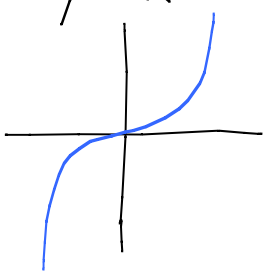
Ex: $y = x$



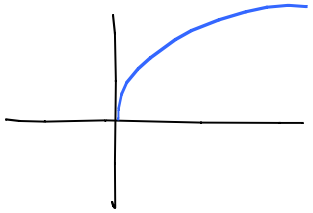
$y = x^2$



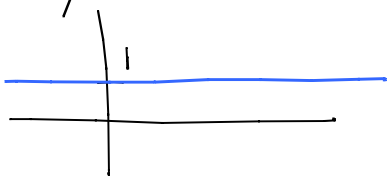
$y = x^3$



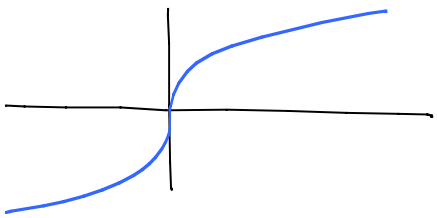
$y = x^{1/2} = \sqrt{x}$



$y = x^0 = 1$



$y = x^{1/3} = \sqrt[3]{x}$



note the domains

Polynomials: finite sums of power functions with non-negative exponents

the highest exponent of a polynomial is its degree

Ex: $y = 3$ deg = 0 constant

$y = 2x + 3$ deg = 1 linear

$y = x^2 + 2x + 3$ deg = 2 quadratic

$y = -x^3 + x^2 + 2x + 3$ deg = 3 cubic

$y = -2x^4 - x^3 + x^2 + 2x + 3$ deg = 4 quartic

$y = -3x^5 - 2x^4 - x^3 + x^2 + 2x + 3$ deg = 5 quintic

Rational Functions: $\frac{P(x)}{Q(x)}$ where P, Q are polynomials

Domain: All real numbers except real roots of Q

Ex: $\frac{x^2+1}{x}$ not defined at $x = 0$

Algebraic Functions:

Any thing that involves only:

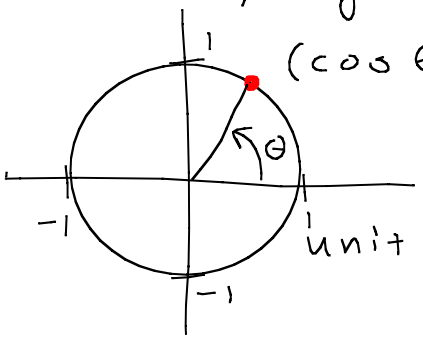
- addition / subtraction
- multiplication
- division
- taking roots

Domain: anywhere the expression is defined

Ex: $y = \sqrt{x^2 - 1}$ Domain: $[-1, 1]$

Trancendental Functions: Anything not Algebraic, including:

Trig fns:



unit circle
$x^2 + y^2 = 1$

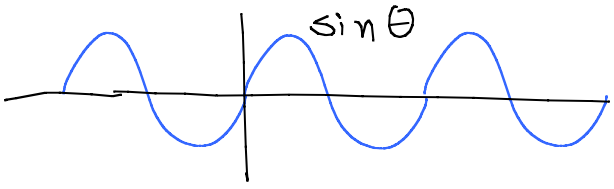
2π radians $= 360^\circ$

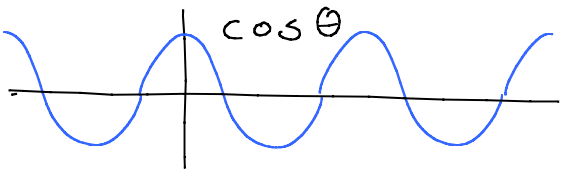
$\sin(\theta + 2\pi) = \sin(\theta)$

$\cos(\theta + 2\pi) = \cos(\theta)$

$\sin(\theta + \pi/2) = \cos(\theta)$

$-1 \leq \cos\theta \\ \sin\theta \leq 1$

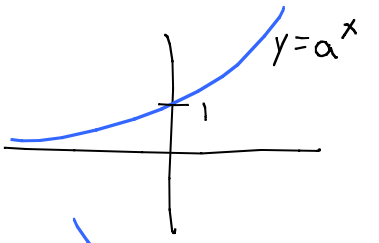




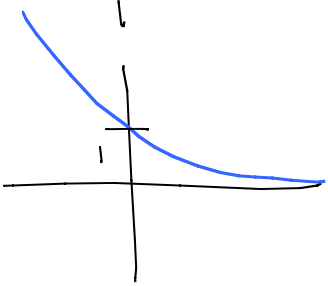
More on these later

Exponential fns: $y = a^x$

in particular $y = e^x$ e = Euler's constant ≈ 2.71



if $a > 1$

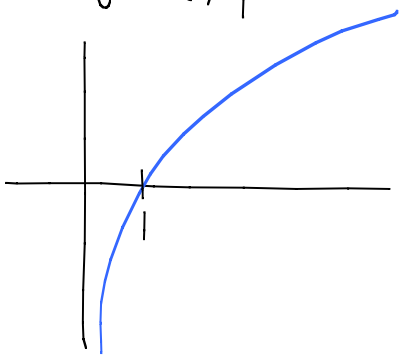


if $0 < a \leq 1$

Domain: $(-\infty, \infty) = \mathbb{R}$
Range: $(0, \infty)$

Used for exponential growth/decay

Logarithmic Fns:
Inverse of exp



$y = \log_a x$

$y = \log x$

log base e