Calc I Lecture 18 - Antiderivatives

Note Title 11/12/2008

Question: If we are given $f(x)$, can we find a function $F(x)$ with $F'(x) = f(x)$?

Such a function F is called the antiderivative of f.

From before:

Theorem: If $F_1'(x) = F_2'(x)$ on an interval (a,b) then $F_1(x) - F_2(x) = c$ on (a,b)

This means that two functions with the same derivative must differ by a constant.

$\Longrightarrow$ Any two antiderivatives F_1, F_2 of f differ by a constant!

$$F_1(x) = F_2(x) + C$$

Ex: Antiderivative of $f(x) = 2x$?

$$\frac{d}{dx}(x^2) = 2x$$

So $F(x) = x^2$ is one antiderivative of $f(x) = 2x$

$\Longrightarrow$ $x^2 + C$ is the general form for the antiderivative of $2x$

Ex: Antiderivative for $f(x) = x^4$?

$\frac{d}{dx}(x^5) = 5x^4$ so $\frac{d}{dx}\left(\frac{1}{5}x^5\right) = \frac{1}{5}5x^4 = x^4$

General form for antiderivative: $F(X) = \frac{1}{5}X^5 + C$

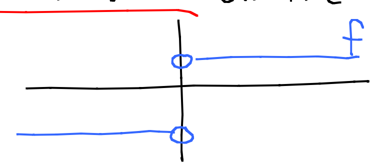
Ex: Antiderivative for $f(x) = \frac{1}{X}$

$\frac{d}{dx}\left(\ln|X|\right) = \frac{1}{X}$ $\longrightarrow$ so on each interval in the domain of f, $F(x) = \ln|x| + C$

The domain has two connected components! $(-\infty, 0)$ & $(0, \infty)$
so Constant may be different on each.

general antiderivative: $F(x) = \begin{cases} \ln|x| + C_1 & x > 0 \\ \ln|x| + C_2 & x < 0 \end{cases}$

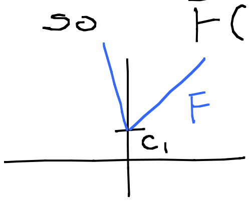
Ex: Find the general form for a continuous antiderivative of $f(x) = \begin{cases} -2 & x < 0 \\ 1 & x > 0 \end{cases}$



General Antiderivative: $F(x) = \begin{cases} -2x + C_1 & x < 0 \\ x + C_2 & x > 0 \end{cases}$

if $F(x)$ continuous $\lim_{x \to 0^-} F(x) = C_2 = \lim_{x \to 0^+} F(x) = C_1$

so $F(x) = \begin{cases} -2x + C_1 & x < 0 \\ x + C_1 & x \geq 0 \end{cases}$ is the general form for a continuous antideriv. of f



Some Antiderivative Rules:

function $f(x)$	Antiderivative $F(x)$
$c\,f(x)$	$c\,F(x)$
$f(x)+g(x)$	$F(x)+G(x)$
$x^n \quad (n \neq -1)$	$\frac{x^{n+1}}{n+1}$
$\frac{1}{x}$	$\ln\lvert x\rvert$
e^x	e^x
$\cos x$	$\sin x$
$\sin x$	$-\cos x$
$(\sec x)^2$	$\tan x$
$\sec x \tan x$	$\sec x$
$1/\sqrt{1-x^2}$	$\arcsin x$
$1/1+x^2$	$\arctan x$

With extra data, we can determine the constants.

Ex: A ball is thrown from the ground into the air at a rate of 10 m/s. When does it land?

Accelleration due to gravity: $-9.8\ m/s^2 = a$

Velocity is antiderivative of accelleration

$$v = -9.8t + C$$

at $t=0$, $v=10$ $\quad -9.8 \cdot 0 + C = 10 \longrightarrow C = 10$

$$v = -9.8t + 10$$

position is antiderivative of velocity

$$P = -\frac{9.8}{2}t^2 + 10t + C$$

at $t=0$, $P=0$ $\quad -\frac{9.8}{2} \cdot 0^2 + 10 \cdot 0 + C = 0 \longrightarrow C = 0$

$$P = -\frac{9.8}{2}t^2 + 10t = t\left(-\frac{9.8}{2}t + 10\right)$$

hit the ground when $P=0$

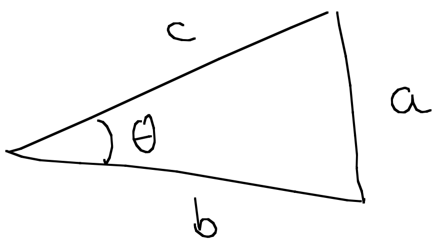
$t\left(-\frac{9.8}{2}t + 10\right) = 0 \longrightarrow t = 0$ or

$t = 20/9.8$

≈ 2.0408 seconds

Hint on HW problem §1.8 #40

law of cosines



$$a^2 = b^2 + c^2 - 2bc \cdot \cos\theta$$