Modern algebra II, spring 2015.

Practice quiz 3

Mark the boxes that are followed by correct statements.

☐ Polynomial $x^{100} - 27x^3 - 6$ is irreducible over $\mathbb{Q}$.
☐ There exists a field of order 100.
☐ There exists an irreducible polynomial of degree 20 over $\mathbb{F}_2$.
☐ Field $\mathbb{F}_8$ is a subfield of $\mathbb{F}_{256}$.
☐ Polynomial $x^6 + x^4 + x^2 + 1$ is irreducible over $\mathbb{F}_4$.
☐ Field $F(x)$, where $x$ is a formal variable and $F$ is a field, admits an automorphism that takes $x$ to $x + 1$.
☐ Field $\mathbb{F}_{128}$ has no subfields other than $\mathbb{F}_2$ and itself.
☐ Product $f(x)g(x)$ is separable iff both $f(x)$ and $g(x)$ are separable.
☐ Polynomial $x^p - 3$ is separable over any field.
☐ Any finite extension of fields in characteristic 0 is simple.
☐ Any finite inseparable extension is simple.
☐ $\text{Gal}(\mathbb{C}/\mathbb{R}) \cong C_2$.
☐ If $E/F$ is Galois and $K$ is an intermediate field, $F \subset K \subset E$, then $E/K$ is Galois.
☐ If $E$ is a finite field and $F$ a subfield, $\text{Gal}(E/F)$ is an abelian group.
☐ If $K/F$ is normal and $E/K$ is normal, then $E/F$ is normal.
☐ $\text{Gal}(\mathbb{Q}(\sqrt{5})/\mathbb{Q}) \cong C_5$.
☐ If $K/F$ is simple and $E/K$ is simple, then $E/F$ is simple.
☐ There exists a degree 3 field extension $E/F$ with $\text{Gal}(E/F)$
a cyclic group of order 6.