Modern algebra II, spring 2015.

Practice Quiz 1

In this quiz we consider only commutative rings.

Mark the boxes that are followed by correct statements.

☐ There are exactly two isomorphism classes of monoids with 2 elements.

☐ If $e$ is an idempotent in a ring $R$, then $1 - e$ is also an idempotent.

☐ The product $e_1 e_2$ of idempotents $e_1, e_2$ in $R$ is an idempotent in $R$. (Does the answer change if we don’t assume that $R$ is commutative?)

☐ Any integral domain is a field.

☐ The set $2 + 4\mathbb{Z}$ is an ideal of $\mathbb{Z}$.

☐ If $a, b$ are in ideal $I$ of $R$, their difference $a - b$ is also in $I$.

☐ Any ideal of $\mathbb{Z}$ is principal.

☐ Any ideal of $\mathbb{Z}/n$ is principal for any $n > 1$.

☐ Any ideal of $\mathbb{Z}/n \times \mathbb{Z}/m$ is principal for any $n, m > 1$.

☐ Rings $\mathbb{Z}/24$ and $\mathbb{Z}/6 \times \mathbb{Z}/4$ are isomorphic.

☐ The set of polynomials without constant term is an ideal of $\mathbb{Q}[x]$.

☐ The set of polynomials whose coefficients sum up to zero is an ideal of $\mathbb{Q}[x]$.

☐ The set of polynomials with constant term an integer divisible by 3 is an ideal of $\mathbb{Z}[x]$.

☐ The set of polynomials of degree at most 3 is an ideal of $\mathbb{Q}[x]$. 
The intersection $I \cap J$ of ideals $I, J$ of ring $R$ is an ideal of $R$.

The union $I \cup J$ of ideals $I, J$ of $R$ is an ideal of $R$.

A field $F$ has only two ideals.

There exists a ring with exactly three ideals.

Automorphisms of a ring constitute a group.

(this one is hard) Any monoid with 3 elements is commutative.