1. (10 points) Mark the boxes that are followed by correct statements.

□   The intersection \( H \cap K \) of normal subgroups \( H \) and \( K \) of \( G \) is normal in \( G \).

□   Any abelian group of order 8 contains an element of order 4.

□   The centralizer of \((12)\) in \( S_3 \) has order 3.

□   Any group of order 10 is abelian.

□   If an abelian group \( G \) has an element of order 3, then \( G \) has at least 3 characters.

2. (10 points) Classify homomorphisms from (a) the cyclic group \( C_4 \) to \( S_3 \), (b) the cyclic group \( C_5 \) to \( S_3 \), (c) the group \( S_3 \) to \( C_2 \). Explain why these are all the homomorphisms.

3. (10 points) For each of the following groups \( G \) determine whether \( H \) is a normal subgroup of \( G \).
   (a) \( G = S_4 \) and \( H \cong S_3 \) is the subgroup of permutations that fix 4.
   (b) \( G = A_4 \) and \( H = \{1, (12)(34)\} \).
   (c) \( G = D_n \), the dihedral group, and \( H = C_n \), the subgroup of rotations in \( G \).

4. (10 points) Write down the character table of the group \( C_2 \times C_2 \).

5. (10 points) Write down a proof that, given two subgroups \( K, H \) of \( G \) with \( K \) normal, the set \( KH = \{kh : k \in K, h \in H\} \)
is a subgroup of $G$.

6. (20 points) (a) Find all subgroups of $Q_8$.
(b) Show that all subgroups of $Q_8$ are normal.
(c) Write down definition of a solvable group.
(d) Use this definition to prove directly that $Q_8$ is solvable (We proved in class that any group of order $p^n$ is solvable; don’t use this result).