Name:

Modern algebra I, Midterm exam 1.

1. Mark the boxes that are followed by correct statements.
   □  $90 \equiv 10 \pmod{15}$
   □  If $K$ is a subgroup of $H$ and $H$ is a subgroup of $G$, then $K$ is a subgroup of $G$.
   □  The symmetric group $S_5$ has order 100.
   □  If $g$ and $h$ commute then $g^{-1}$ and $h^{-1}$ commute.
   □  Any group of order 71 is abelian.

2. Suppose that $f : A \rightarrow A$ is a surjective map from a finite set $A$ to itself. Prove that $f$ is bijective.

3. Consider the group $\mathbb{Z}_{10}^*$ of invertible residues modulo 10. List all elements of this group, find the order of the group and the order of every element. Draw the multiplication table for the group. Is the group cyclic? List all subgroups of this group.

4. Simplify the following permutations:
   (a) $(123)(132)(14)(23)$, (b) $(153)(243)(1432)$, (c) $(321)^2(14)^{-3}$, (d) $(1342)^{10}$, (e) $((13)(123))^4$.

5. (a) Elements $a, b, c$ in a group $G$ satisfy $abc = 1$. Show that $bca = 1$.
   (b) Elements $a, b \in G$ satisfy $a^4b^3 = 1$ and $b^2a^2 = 1$. Express $b$ as a power of $a$.

6. Let $x, y \in \mathbb{N}$ be relatively prime. If $xy$ is a perfect square (that is, $xy = n^2$ for some $n \in \mathbb{N}$), prove that $x$ and $y$ must both be perfect squares.