Modern Algebra I, Fall 2014

Homework 8, due Wednesday November 12 before class.

1. List isomorphism classes of abelian groups of orders (a) 16, (b) 40, (c) 125.

2. Do parts (2),(3), and (6) of Exercise 5 on page 12.6 of Gallagher’s notes (Section 12 on Isomorphism theorems and solvable groups).

3. Prove that the dihedral group $D_n$ of order $2n$ is solvable.

4. Dihedral group $D_4$ is the group of symmetries of the square. Describe orbits and stabilizer subgroups for various points in the square. Can you realize the rotation subgroup $C_4$ of $D_4$ as the stabilizer of a point?

5. Find the conjugacy classes and the class equation for each of the following groups
   (a) $S_4$, (b) $D_5$, (c) $\mathbb{Z}_6$, (d) $Q_8$.

6. Let $G$ act on a set $X$. Given $A \subset X$ we can define $gA = \{ ga : a \in A \}$. Thus, $gA$ is a subset of $X$. Check that this is an action of $G$ on the powerset $P(X)$ of $X$ (the set of all subsets of $X$). What is the stabilizer of the empty set $\emptyset$ under this action? What is the stabilizer of a one-element set $\{x\}$, for $x \in X$?

7. Let $G$ be an abelian group of order $m$. If $n$ divides $m$, prove that $G$ has a subgroup of order $n$ (Hint: use the classification theorem for finite abelian groups.)