Group theory

Quiz: testing for group structure

Which of the following pairs \((X, \ast)\), where \(X\) is a set and \(\ast\) a binary operation are groups? Mark corresponding squares.

1. □ The empty set \(\emptyset\) equipped with the only possible binary operation \(\emptyset \times \emptyset \to \emptyset\).
2. □ \((\mathbb{Z}, +)\). The set of all integers \(\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}\) with binary operation addition.
3. □ \((\mathbb{Z}, -)\). Integers with binary operation subtraction.
4. □ \((\mathbb{Z}, \ast)\). Integers with binary operation multiplication.
5. □ \((\mathbb{Q}, +)\). The set of all rational numbers, with addition as binary operation.
6. □ \((\mathbb{Q}, \text{max})\). All rational numbers, binary operation max gives the maximum of the two numbers.
7. □ \((\mathbb{Q}, \bullet)\), where \(x \bullet y = x + 2y\).
8. □ \((\mathbb{Q}^*, \ast)\). Here \(\mathbb{Q}^* = \mathbb{Q} \setminus \{0\}\) is the set of all nonzero rational numbers and the binary operation is multiplication.
9. □ \((\mathbb{Q}^*, /)\). All nonzero rationals, binary operation division.
10. □ \((\mathbb{R}_{>0}, \ast)\). All positive real numbers, binary operation multiplication. Here by a positive number we mean a real number strictly greater than 0.
11. □ \((\mathbb{R}_{>0}, +)\). All (strictly) positive real numbers, addition as binary operation.
12. □ \((\mathbb{R}^*, \ast)\). All nonzero real numbers, binary operation multiplication.
13. □ \((\mathbb{Q}, \bullet)\). All rational numbers, binary operation \(x \bullet y =\)
\[ x + y - xy. \]