Name: ________________________ UNI: ________________

Modern algebra I, Spring 2017. Midterm exam 2.

Notes, textbooks, cell phones are not allowed on the exam. Please write your name on each blue book that you use. You can solve problems in any order. Please return your exam (with your name on it) along with your blue book. Good luck!

1. (15 points) Simplify the following permutations and write them as products of disjoint cycles:
(a) $(132)(14)(13)$, (b) $(12)(13)(12)(13)$, (c) $(1532)^{11}$.
For each permutation (a)-(c), determine whether it's odd or even.

2. (10 points) For each of the following groups G determine whether H is a normal subgroup of G. Give a brief justification.

(a) $G = S_4$ and $H \cong S_3$ is the subgroup of permutations that fix 4.

(b) $G = \mathbb{Z}$ and $H = 5\mathbb{Z}$.

3. (15 points) Describe all homomorphisms from (a) the cyclic group C_2 to S_3, (b) the cyclic group C_5 to S_3, (c) the cyclic group C_2 to C_2.

4. (10 points) Write down the character table of the cyclic group C_4.

5. (15 points) List all isomorphism classes of abelian groups of order (a) 24, (b) 14, (c) 36.

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7. (15 points) Mark the boxes that are followed by correct statements. You can write short justifications in the spaces below the statements.

☐ The image $\psi(G)$ of any homomorphism $\psi : G \longrightarrow H$ is a normal subgroup of H.

☐ Any abelian group of order 8 contains an element of order 4.

☐ A group of order 40 can have a subgroup of order 12.

☐ Any group of order 10 is abelian.

☐ The symmetric group S_4 has trivial center.

Extra credit: Prove that any homomorphism from the group $C_n \times C_n$ for $n > 1$ to the circle group $\mathbb{T}$ has a nontrivial kernel.