Name: ______________________ Uni: ____________

Modern algebra I, Spring 2017. Midterm exam 1.

Notes, textbooks, calculators or other computing devices are not allowed on the exam. Please write your name on each blue book that you use. Label each problem clearly as you answer it. You can solve problems in any order. You must return your exam (with your name on it) along with your blue book. Good luck!

1.(10 points) Mark the boxes that are followed by correct statements.

☐ $(A \cup B) \setminus B = A \setminus B$ for any two sets A and B.

☐ $A \setminus (A \setminus B) = B$ for any two sets A and B.

☐ $75 \equiv -10 \pmod{17}$

☐ $n \sim m$ iff $5 \mid n - m$ is an equivalence relation on the set $\mathbb{Z}$ of integers.

☐ $gcd(n, m) = gcd(m, n)$ for any natural numbers n, m.

2.(10 points) Let sets $A = \{a, b, c\}$ and $B = \{d, e\}$. Give an example of a map $f : A \longrightarrow B$ that is not surjective.

3.(10 points) Show that the set of all vectors in the plane $\mathbb{R}^2$ constitute a group under addition of vectors. What is the unit element of this group? What is the inverse of a vector?

4.(15 points) Consider the group $\mathbb{Z}_{10}^*$ of invertible residues modulo 10. List all elements of this group, find the order of the group and the order of every element. Draw the multiplication table for the group. Is the group cyclic?

5. (a) (5 points) State the definition of a subgroup H of a group G.

(b) (10 points) Let $H = \{2^k : k \in \mathbb{Z}\}$. Prove that H is a subgroup of $\mathbb{Q}^*$. ($\mathbb{Q}^*$ is the group of nonzero rational numbers under multiplication).

6.(10 points) Elements a and b in a group G satisfy $ab^2a = 1$. Show that $b^2 = a^{-2}$.

Extra credit: Give an example of a subgroup of $\mathbb{C}^*$ of order 3.