Modern Algebra I, Spring 2017

Homework 6, due Wednesday March 1 before class.
Read the rest of Section 5.1 (starting with Transpositions) and Sections 6.1-6.3. Solve the following problems.

1. Express the following permutations as products of transpositions and identify them as even or odd.
(a) (15326), (b) $(142)(356)(78)$, (c) $(1536)(79428)$.

2. Find all possible orders of elements of (a) S_4, (b) A_4, (c) S_5, (d) A_5. (Hint: what are possible cycle types of permutations in these groups?)

3. Give an example of an element of A_{10} of order 15.

4. If σ is a cycle of odd length, prove that σ^2 is also a cycle.

5. (a) List left cosets of the subgroup $\langle 5 \rangle$ of $\mathbb{Z}/15$. Are left cosets equal to right cosets for this subgroup? What is the index of this subgroup?

(b) List left and right cosets of the subgroup $H = \{1, (23)\}$ in the symmetric group S_3. (We did a very similar example in class.) Which left cosets are not right cosets?

6. Let G be a cyclic group of order n. Show that there are exactly $\phi(n)$ generators for G. Here ϕ is the Euler's function, see Section 6.3.

7. (a) Find all subgroups of S_3. Don't forget the trivial group and S_3 itself.

(b) Find all subgroups of A_4. (You can use that A_4 has no subgroup of order 6, this is Proposition 6.15 in Judson. Also, check that $\{id, (12)(34), (13)(24), (14)(23)\}$ is a subgroup of order 4. If you label vertices of a rectangle by $1, 2, 3, 4$, these permutations will give you exactly all four symmetries of a rectangle.)

(c) Find all subgroups of $\mathbb{Z}/3 \times \mathbb{Z}/3$ (this group is the direct product of two copies of $\mathbb{Z}/3$).

To find all subgroups in (a)-(c), use that the order of a subgroup divides the order of the group. Also use that any subgroup of prime order is cyclic, generated by some element.