Modern Algebra I, Spring 2017

Homework 3, due Wednesday February 8 before class.
Read Judson Chapter 3 and Section 4.1. Solve the following problems.

I. Write down the Cayley table for the group $\mathbb{Z}/5$. Remember that the group operation is addition. Is your table symmetric?

Exercises 2bd, 7, 27, 33, 45, 49 from Judson, Chapter 3, pages 39-42 (use the latest online edition, 2016).

For 2bd, you'll need to see if there's the unit element in the table. Then either look for a simple reason why a given table is not the Cayley table for any group, or eventually recognize the table as that of a familiar group. (It's too tedious to check associativity of multiplication for all triples a, b, c of elements.)
Solving exercise 7, first check that the operation is associative, then find the unit element for this operation, and then find the inverse of a, for any a. Also check commutativity.

II. Prove that if H is a subgroup of K, and K is a subgroup of G, the H is a subgroup of G.
III. Find all subgroups of the Klein four group V_4. (Don't forget the trivial subgroup and V_4 itself.)