Modern Algebra I, Spring 2017

Homework 2, due Wednesday February 1 before class.
This homework covers equivalence relations (end of Chapter 1 in Judson, also covered in Gallagher), and integer arithmetic (Judson Chapter 2 and Gallagher Sections 1, 2).

1. Which of the following relations R on sets X are equivalence relations? Justify your answer (for each relation, check whether it's reflexive, symmetric, and transitive).
1) $X = \mathbb{Q}$, $(a, b) \in R$ iff $a \leq b$.
2) $X = \mathbb{R}$, $(a, b) \in R$ iff $a - b \in \mathbb{Z}$.
3) $X = \mathbb{Z}$, $(a, b) \in R$ if $a + b$ is odd.
4) X is any set, $(a, b) \in R$ iff $a = b$.
5) $X = \mathbb{C}$, $(z, w) \in R$ iff $|z| = |w|$ (here $|z|$ denotes the absolute value of complex number z).
6) $X = \mathbb{Q}$, $(n, m) \in R$ iff $n = m$ or $n = -m$.

2. Compute $gcd(-100, 16), gcd(468, 528)$, $gcd(-30, -27)$, $gcd(-15, 0)$, $gcd(1, -1)$, $lcm(100, 16), lcm(27, -5)$.

3. Let $x, y \in \mathbb{N}$ be relatively prime. If xy is a perfect square, prove that x and y must both be perfect squares. (Hint: use the Fundamental Theorem of Arithmetic aka Unique Factorization Theorem.)

Read Gallagher §1. Divisors, GCD, prime numbers and solve exercise 1 on page 1.2 and exercise 2 on page 1.3. Read Gallagher §2. Unique factorization and solve exercise 1 on page 2.4.