Modern Algebra I, Spring 2017

Homework 12, due Wednesday April 19 before class.

Read Gallagher Section 18, which contains proofs of the Sylow theorems (alternatively, read Judson Section 15.1). Also read Judson Section 15.2 (up to but not including Theorem 15.13) for some applications of the Sylow theorems.

1. Classify all groups of order (a) 35 , (b) 59, (c) 77, (d) 26, (e) 325 up to isomorphism. Justify your answers using Sylow theorems and other results proved in class.

2. If H is a normal subgroup of a finite group G and $|H| = p^k$ for some prime p, show that H is contained in every Sylow p-subgroup of G.

3. What are the orders of Sylow p-subgroups of A_4, for $p = 2, 3, 5$? For each of these p, give an example of a Sylow p-subgroup of A_4. Which of your examples are normal subgroups of A_4?

4. What is the order of a Sylow p-subgroup of the symmetric group S_5 for $p = 2, 3, 5$? For each of these p, give an example of a Sylow p-subgroup of S_5. (Use the solution to the same problem for S_4, done in class, as a blueprint.)

5. Show that every group of order 45 has a normal subgroup of order 9.

6. Suppose that G is a finite group of order $p^n k$, where $k < p$, and p a prime. Show that G must contain a normal subgroup.