Lie groups

Homework #2, due Wednesday, February 21.

1. Use the degree formula for an irreducible representation $V(\lambda)$ to compute the dimension of an irreducible representation of $sl(n)$ with the highest weight $\sum_{i=1}^{n-1} m_i\lambda_i$ as a function of $m_1, \ldots, m_{n-1}$.

2. Classify highest weight vectors in the symmetric powers S^kV of the fundamental n-dimensional irreducible representation of $sl(n)$ and use this to show that S^kV is irreducible.

3. Draw the weight diagram (with weight multiplicities) of the symmetric cube S^3V of the fundamental representation V of $sl(3)$.

4. Let V be the fundamental and L the adjoint representation of $sl(3)$. Determine how tensor products $V \otimes V$ and $V \otimes L$ decompose into direct sums of irreducible representations, using the method discussed in class (see also exercise 9 on page 142 of Humphreys' book). Do the same for $S^2V \otimes S^2(V^*)$.

5. Let $V(\lambda)$ be an irreducible representation of $so(2n)$ (the root system is D_n). What is the highest weight of $V(\lambda)^*$?

6. Let α_1, α_2 be simple roots of B_2, with α_1 the longer root. Draw the weight lattice together with α_1, α_2 and the fundamental weights λ_1, λ_2. Find dimensions and weight spaces of fundamental representations $V(\lambda_1)$, $V(\lambda_2)$. Determine multiplicities of irreducible representations in tensor products $V(\lambda_1)^{\otimes 2}$, $V(\lambda_1) \otimes V(2\lambda_2)$, $V(\lambda_1+\lambda_2) \otimes V(\lambda_1)$, $V(3\lambda_1) \otimes V(\lambda_2)$.