Introduction to knot theory, Spring 2013

Homework 6, due Thursday, March 7

1. (a) Let $R = \text{Mat}(n, F)$ be the algebra of $n$ by $n$ matrices with coefficients in a field $F$. Show that any trace-like map $\epsilon : R \rightarrow F$ (an $F$-linear map with $\epsilon(ab) = \epsilon(ba)$ for all $a, b \in R$) is a multiple of the usual trace map on matrices.

(b) Classify all trace-like maps from $TL_2$ and $TL_3$ into the ground field $\mathbb{Q}(A)$ (use the idea about closing an element of $TL_n$ into a diagram on a punctured plane and keeping track of circles that go around the puncture).

(b) The element $u_2u_1$ generates a left ideal $TL_3u_2u_1$. Determine the dimension of this ideal as a vector space over the ground field $\mathbb{Q}(A)$. This ideal is a left module over $TL_3$. Prove that this module is isomorphic to the module $V_{3,1}$ over $TL_3$ (Recall that in class we defined modules $V_{n,k}$ over $TL_n$ for $k = n, n-2, ...$). Show that this module is simple.

2. In class we constructed a homomorphism $\psi : B_n \rightarrow TL_n^\ast$. Show that the image $\psi(\theta_n)$ of the central braid $\theta_n = \Delta_n^2$ is central in $TL_n$. Compute $\psi(\theta_3)$ in the monomial basis of $TL_3$ (Hint: first compute $\psi(\Delta_3)$).

3. Draw a non-alternating diagram $D$ with at least 5 crossings of some knot (preferably not the unknot). Determine diagrams $s_+D$ and $s_-D$. What are the highest and lowest powers of $A$ that these diagrams contribute to $<D>$? Do these powers survive in $<D>$ or get cancelled out?