

Algebraic topology, Spring 2014

Homework 5, due Wednesday, May 7

1. Generalize proposition 1.11 (Hatcher, page 23) from \mathbb{S}^k to the suspension SX of a finite CW-complex X . Explain how the proof changes.
2. Classify complex vector bundles over (a) $X = \mathbb{S}^2 \vee \mathbb{S}^1$, (b) $X = \mathbb{S}^2 \vee \mathbb{S}^2$, (c) X a closed oriented surface of genus g , (d) X a finite bouquet of circles. Determine the ring $K(X) = K^0(X)$ for X in (a)-(d) and its ideal $\tilde{K}(X)$.
3. Describe $\tilde{K}^*(X \sqcup Y)$ in terms of $\tilde{K}^*(X)$ and $\tilde{K}^*(Y)$.
4. Go through the proof of Lemma 2.18 (Hatcher, page 61) and preliminary material (starting on page 59). Modify the proof of Lemma 2.18 to get analogous result for cohomology with integer coefficients instead of K-theory.
5. In the proof of multiplicativity of Adams operation ϕ^k on page 64 (2nd paragraph from the top) one step is skipped. Which one? Fill it in.