

Algebraic topology, Spring 2014

Homework 4, due Wednesday, April 23

1. Show that countable abelian groups form a Serre class.
2. Prove Case 3 of Lemma 1.19 in Hatcher Spectral Sequences notes (Section 1.1 page 15).
3. Let $X = S^2 \vee S^2$. Explain why all homotopy groups of X are finitely generated. Determine ranks of homotopy groups π_3 and π_4 of X . (Hint: first compute $\pi_2(X)$ and consider the fibration $K(\pi_2(X), 1) \rightarrow X|_3 \rightarrow X$. Use it to determine rational homology of $X|_3$ and rank of $\pi_3(X)$. Then likewise determine some rational homology groups of $X|_4$.) Can you use this approach to determine ranks of $\pi_n(X)$ for $n > 4$?
4. Exercise 3 in Hatcher SS notes Section 1.2 page 51.