

## Algebraic topology, Spring 2014

### Homework 3, due Monday, March 24

Read Section 1.1 of Hatcher's notes on spectral sequences (you can skip Serre classes subsection for now).

Exercises 1, 2 for Section 1.1 (page 23).

3. Write down all pages of the spectral sequence for the Hopf fibration  $\mathbb{S}^1 \rightarrow \mathbb{S}^{2n+1} \rightarrow \mathbb{C}\mathbb{P}^n$ . At what page does the sequence degenerate?

4. Read example 1.11 in Hatcher. Explain how this example generalizes if we replace "multiplication by 2" by "multiplication by  $k$ " for  $k > 2$ .

5. Give an example of a filtered complex of vector spaces whose associated spectral sequence degenerates at (a)  $E^1$ , (b)  $E^2$ , (c)  $E^n$ .

6. (a) Give an example of a filtered complex of free abelian groups which has a single nontrivial homology group  $\mathbb{Z}/4$  and whose spectral sequence has  $E^\infty$ -term  $\mathbb{Z}/2 \oplus \mathbb{Z}/2$  (in some bidegrees). Is there an example when these two  $\mathbb{Z}/2$ 's are positioned at a given distance  $k > 0$  from each other along the diagonal where they live?

(b) generalize from  $\mathbb{Z}/4$  to  $\mathbb{Z}/2^n$ , such that the  $E^\infty$  term is  $(\mathbb{Z}/2)^n$ .