Algebraic topology, Spring 2014

Homework 1, due Wednesday, February 5

1. For abelian groups G, H and $n > 0$ show that cohomological operations

$$\theta \ : \ H^n(*, G) \longrightarrow H^0(*, H)$$

are classified by elements of H. Explain how this operation acts on elements of $H^n(X, G)$ (hint: consider path-connected components of X). Specialize to $G = H = \mathbb{Z}/2$. What does the only nontrivial cohomological operation do? Is it linear?

2. Classify cohomological operations

$$\theta \ : \ H^0(*, G) \longrightarrow H^n(*, H)$$

(consider cases $n = 0$ and $n > 0$ separately).

3. Given a cohomological operation $\theta : H^n(*, G) \longrightarrow H^m(*, H)$ and $m > 0$, prove that $\theta_X(0) = 0$ for any X.

4. Classify cohomological operations

$$H^1(*, \mathbb{Z}) \longrightarrow H^n(*, \mathbb{Z}), \quad n \geq 1.$$

5. Classify cohomological operations

$$H^1(*, \mathbb{Z}/2) \longrightarrow H^n(*, \mathbb{Z}/2), \quad n \geq 1.$$

How many operations are there for a given n? How do they act on an arbitrary element $\alpha \in H^1(X, \mathbb{Z}/2)$? Which ones are linear?

6. Classify cohomological operations

$$H^2(*, \mathbb{Z}) \longrightarrow H^n(*, G)$$

for $n > 0$, where the group G is (a) $\mathbb{Z}$, (b) $\mathbb{Z}/2$, (c) $\mathbb{Q}$. Determine which operations are linear.