Homework 4, due Wednesday, October 2

From Hatcher:
Section 4.1 exercises 11, 14 (pages 358-359).

1. (a) Prove that the long sequence of a pair \((X, A)\)

\[
\ldots \rightarrow \pi_n(A) \xrightarrow{i_*} \pi_n(X) \xrightarrow{j_*} \pi_n(\pi_n(X, A)) \xrightarrow{\partial} \pi_{n-1}(A) \rightarrow \ldots
\]

is exact.

(b) Assuming \(A\) is a retract of \(X\), show that \(i_*\) is a monomorphism, \(j_*\) is an epimorphism, \(\partial = 0\) and \(\pi_n(X) = \pi_n(X, A) \oplus \pi_n(A)\).

(c) Do a similar analysis for the case when \(A\) is contractible to a point in \(X\).

2. Prove the 5-lemma (in the case when maps \(f_1, f_2, f_4, f_5\) are isomorphisms).

3. Using the fiber bundle \(S^\infty \rightarrow \mathbb{C}P^\infty\) with fiber \(S^1\) compute homotopy groups of \(\mathbb{C}P^\infty\) and explain why it is an Eilenberg-Maclane space.