Algebraic topology, Fall 2013

Homework 3, due Wednesday, September 25

From Hatcher:
Section 1.2 exercises 15, 16 (page 54).
Section 1.3 exercises 5, 9, 10 (pages 79-80).
Section 1.4 exercises 2, 12 (pages 358-359).
1. Describe explicitly a universal covering space of the subspace of \( \mathbb{R}^3 \) that is the union of a sphere and a diameter.
2. Prove that if CW complex \( X \) has no cells of dimension 1, 2, \ldots, \( n \), then \( \pi_i(X) = 0 \) for \( i \leq n \).
3. (a) Assuming that \( \pi_2(S^2) = \mathbb{Z} \) compute \( \pi_3(S^2 \vee S^2) \) (hint: embed this space into a certain larger space). (b) Can you determine \( \pi_2 \) of the wedge \( S^2 \vee S^1 \)?

Discussion. Sample problems: Section 1.3 exercises 6, 20.
1. Show that the join \( X \ast Y \) of two nonempty spaces \( X \) and \( Y \) is simply-connected if \( X \) is path-connected.