Representations of finite groups

Homework #8, due Wednesday, November 11.

1. The quaternion group Q_8 has 5 irreducible representations (the character table was derived in class). Let V be the unique 2-dimensional irreducible representation of Q_8.

(a) Find the characters of $\Lambda^2(V)$ and $S^2(V)$. Is any of these representations irreducible? What are the multiplicities of irreducible representations in $\Lambda^2(V)$ and $S^2(V)$?

(b) Let W be the 4-dimensional representation of Q_8 which is the direct sum of all 1-dimensional irreducibles of Q_8. What's the character of W? Decompose $\Lambda^2(W)$ into direct sum of irreducible representations of Q_8.

2. Show that the direct sum of two self-dual representations is self-dual.

3. Let V, W be irreducible representations of a finite group G. Prove that $V^* \otimes W$ contains the trivial representation with multiplicity at most 1. Show that the multiplicity is one if and only if V and W are isomorphic.

4. Let z be a central element of a finite group G and V an irreducible representation of G. Show that z acts on V as a multiple of the identity endomorphism (hint: use Schur's lemma).

5. Give an example of a partial order on a 5-element set which has exactly two maximal elements. Is your partial order a chain? How many minimal elements does you order have? Describe all chains (totally ordered subsets) in your partial order. Choose a 2-element subset of your set and list all upper bounds for it.

6. Give an example of a category which has exactly 3 objects and 6 morphisms.