Representations of finite groups

Homework #7, due Wednesday, October 28.

1. Show that for any representation V of a group G representation $V \otimes \underline{\mathbb{C}}$ is isomorphic to V, where $\underline{\mathbb{C}}$ is the trivial representation of G.

2. We classified irreducible representations of S_4 in class and denoted them V_0, V_1, V_2, V_3, V_4. Determine multiplicities of irreducible representations in various tensor products $V_i \otimes V_j$, for $i, j = 0, 1, 2, 3, 4$. You can utilize symmetries of tensor products and several tricks discussed in class.

3. The quaternion group Q_8 has a unique irreducible 2-dimensional representation V. What are the multiplicities of irreducible representations in $V \otimes V$? In $V \otimes V \otimes V$?

4. (a) Let V, W, K be vector spaces. Show that $(V \oplus W) \otimes K$ and $(V \otimes K) \oplus (W \otimes K)$ are isomorphic.

(b) Assume that V, W, K are, in addition, representations of G. Check whether the isomorphism you constructed above extends to an isomorphism of representations.

5. (a) Show that the trivial representation of any group is self-dual.

(b) For each irreducible representation of $\mathbb{Z}/4$ determine its dual. How many irreducible representations of $\mathbb{Z}/4$ are self-dual?

(c) Among the groups listed below select those with every irreducible presentation being self-dual:

$$\mathbb{Z}/2, \qquad S_4, \qquad Q_8, \qquad \mathbb{Z}_6, \qquad \mathbb{Z}/2 \times \mathbb{Z}/2.$$

6*(optional). Let V be a finite-dimensional representation of a group G. Prove that V^* is irreducible if and only if V is.