

## Representations of finite groups

**Homework #7**, due Monday, April 3.

1. Check that  $\mathbb{C}G$ -modules constitute a category. Its objects are  $\mathbb{C}G$ -modules and the morphisms are module maps. What is the initial object in this category?
2. (a) Give an example of a category with 3 objects and 6 morphisms. Does your category have an initial object? A terminal object?  
(b) Same problem, but for a category with 3 objects and 5 morphisms.
3. Objects  $a, b$  of a category  $C$  are called *isomorphic* if there exists morphisms  $x : a \rightarrow b$  and  $y : b \rightarrow a$  such that  $yx = \text{Id}_a$  and  $xy = \text{Id}_b$ . Show that isomorphism is an equivalence relation on objects of a category. Give an example of a category with infinitely many objects but such that all objects are isomorphic.