

Representations of finite groups

Homework #5, due Monday, March 6.

1. Exercise 2 on page 166 (justify your answer).

2. Let f be the function on the conjugacy classes of the symmetric group S_3 given by $f(1) = 4$, $f((12)) = 0$, $f((123)) = 1$. Determine whether or not f is the character of a representation of S_3 . Do the same for the function $h(1) = 6$, $h((12)) = 2$, $h((123)) = -3$.

3. The quaternion group Q_8 has 8 elements $\pm 1, \pm i, \pm j, \pm k$ and the multiplication $i^2 = j^2 = k^2 = -1$, $ij = -ji = k$, $jk = -kj = i$, $ki = -ik = j$.
 - (a) Find the conjugacy classes of Q_8 and the commutator subgroup $[Q_8, Q_8]$.
 - (b) Use this information to classify one-dimensional irreducible representations of Q_8 .
 - (c) How many irreducible representations does Q_8 have?
 - (d) Construct a two-dimensional irreducible representation of Q_8 by using $1, j$ as a basis of \mathbb{C}^2 and making Q_8 act by left multiplication on this basis.
 - (e) Write down the character table of Q_8 .

4. Let V be the permutation representation of S_3 and W the sign representation of the same group. Determine $\langle V, V \rangle$, $\langle V, W \rangle$ and $\langle W, W \rangle$.