**Representations of finite groups**

**Homework #5,** due Wednesday, October 14.

1. Describe all irreducible representations of $\mathbb{Z}/2 \times \mathbb{Z}/2$.

2. Verify orthogonality relations for characters of irreducible representations of cyclic groups $\mathbb{Z}/2$ and $\mathbb{Z}/3$.

3. Check orthogonality relations for characters of irreps of $S_3$ using the character table derived in class.

4. Let $V$ be the 2-dimensional complex representation of $\mathbb{Z}/3$ where the generator $g$ of $\mathbb{Z}/3$ acts on the basis vectors by

$$g(v_1) = v_2, \qquad g(v_2) = -v_1 - v_2.$$

Express $V$ explicitly as a direct sum of irreducible representations. Compute $\chi_V$.

5. Prove that a finite group $G$ is abelian if every complex irreducible representation of $G$ is one-dimensional.

6. Prove that the character $\chi_V$ is multiplicative,

$$\chi_V(gh) = \chi_V(g)\chi_V(h), \qquad \forall g, h \in G,$$

if and only if $V$ is one-dimensional.