Representations of finite groups

Homework #11, due Wednesday, December 2.

1. (a) If $H \subset G$ is a subgroup of a finite group G, prove that any irreducible representation W_i of G appears as a direct summand of $\mathrm{Ind}(V_j)$, for some irreducible representation V_j of H (choice of j might depend on i).

(b) Suppose that a finite group G has an abelian subgroup H of index 2. Show that any irreducible representation of G is at most two-dimensional (hint: apply part (a)). Conclude that any irreducible representation of dihedral or binary dihedral group is at most two-dimensional.

2. Explain why the quaternion groups Q_8 is one of the binary dihedral groups D^*_{2n}. Which one?

3. Show that the group G with generators a, b and defining relations $a^2 = b^2 = (ab)^2$ is isomorphic to D^*_4.