Representations of finite groups

Homework #11, due Wednesday, December 2.

1. (a) If $H \subset G$ is a subgroup of a finite group $G$, prove that any irreducible representation $W_i$ of $G$ appears as a direct summand of $\text{Ind}(V_j)$, for some irreducible representation $V_j$ of $H$ (choice of $j$ might depend on $i$).

(b) Suppose that a finite group $G$ has an abelian subgroup $H$ of index 2. Show that any irreducible representation of $G$ is at most two-dimensional (hint: apply part (a)). Conclude that any irreducible representation of dihedral or binary dihedral group is at most two-dimensional.

2. Explain why the quaternion groups $Q_8$ is one of the binary dihedral groups $D_{2n}^*$. Which one?

3. Show that the group $G$ with generators $a, b$ and defining relations $a^2 = b^2 = (ab)^2$ is isomorphic to $D_4^*$. 