

## Representations of finite groups

**Homework #10**, due Wednesday, May 3.

1. Suppose that a finite group  $G$  has an abelian subgroup of index  $n$ . Prove that any irreducible representation of  $G$  has dimension at most  $n$ .
2. Start with irreducible representations of the dihedral group  $D_4$  and restrict them to the Klein four group  $V_4$ , which is naturally a subgroup of  $D_4$ . Write down the characters of restricted representations. How do these representations decompose into irreducibles?
3. Consider the inclusion of groups  $A_3 \subset S_3$ . Determine multiplicities of irreducible representations of  $S_3$  in the representations induced from irreducible representations of  $A_3$  (start with the character tables of  $A_3$  and  $S_3$ ). Write down the Frobenius matrix.
4. Do the same for the pair of groups  $S_3 \subset S_4$ .