**Introduction to algebraic topology, Spring 2013**

**Homework 1, due Tuesday, January 29**

1. Show that homotopy is compatible with composition. If $f, g : X \longrightarrow Y$ are homotopic and $f', g' : Y \longrightarrow Z$ are homotopic, then $f'f, g'g : X \longrightarrow Z$ are homotopic.

2. Prove that if $X$ is contractible then $X$ is path-connected.

3. (a) If a set is given indiscrete topology (the only open sets are the empty set and the entire set), the resulting topological space is contractible.

(b) What can you say about a topological space if it's both discrete and contractible?

4. Prove that $X \times Y$ is contractible if and only if both $X$ and $Y$ are contractible.

5. Construct an explicit deformation retraction of the torus with one point deleted onto a graph consisting of two circles intersecting in a point (longitude and meridian circles of the torus).

6. Show that a retract of a contractible space is contractible.

(Exercises 5 and 6 are from Hatcher).