Homework 1, due Tuesday, January 29

1. Show that homotopy is compatible with composition. If \( f, g : X \rightarrow Y \) are homotopic and \( f', g' : Y \rightarrow Z \) are homotopic, then \( f'f, g'g : X \rightarrow Z \) are homotopic.

2. Prove that if \( X \) is contractible then \( X \) is path-connected.

3. (a) If a set is given indiscrete topology (the only open sets are the empty set and the entire set), the resulting topological space is contractible.
   (b) What can you say about a topological space if it’s both discrete and contractible?

4. Prove that \( X \times Y \) is contractible if and only if both \( X \) and \( Y \) are contractible.

5. Construct an explicit deformation retraction of the torus with one point deleted onto a graph consisting of two circles intersecting in a point (longitude and meridian circles of the torus).

6. Show that a retract of a contractible space is contractible.

(Exercises 5 and 6 are from Hatcher).