

## Algebraic topology

**Discussion 4** Section 4.1 of Hatcher (not including the last subsection “CW approximation”).

**Homework 4.** Due Monday, October 9.

1. Show that  $\pi_n(X, A)$  is abelian if  $n \geq 3$ .
2. Prove directly that  $\pi_n(X, A) \cong \pi_{n-1}(A)$  if  $X$  is contractible.
3. Show that the image of the homomorphism

$$\pi_2(X, x_0) \longrightarrow \pi_2(X, A; x_0)$$

in the homotopy long exact sequence of the pair  $(X, A)$  lies in the center of  $\pi_2(X, A; x_0)$ .

4. If  $A$  is a retract of  $X$ , the maps in the homotopy long exact sequence have the following properties:

$\pi_n(A) \rightarrow \pi_n(X)$  is a monomorphism,

$\pi_n(X) \rightarrow \pi_n(X, A)$  is surjective,

$\pi_n(X, A) \rightarrow \pi_{n-1}(A)$  is the zero map.