Complex Homework I

1. Write in the form \( a + bi \):
   
   (a) \((2 + i) - (3 + i)\);
   (b) \(\left(\frac{2}{3} + \frac{1}{4} i\right) + \left(\frac{3}{2} + i\right)\);
   (c) \((1 + 4i)(2 + 4i)\);
   (d) \((2 - 3i)(2 + 3i)\).

2. Write in the form \( a + bi \):
   
   (a) \(\frac{2 + i}{3 + i}\);
   (b) \(\frac{1 + 4i}{2 + 8i}\);
   (c) \(\frac{2 - 3i}{3 + 2i}\).

3. Write in polar form:
   
   (a) \(1 - \sqrt{3}i\);
   (b) \(-5 + 5i\);
   (c) \(\pi i\).

4. Write in the form \( a + bi \) (using De Moivre’s Theorem):
   
   (a) \((\sqrt{3} - i)^7\);
   (b) \((1 + i)^9\).

5. Write in the form \( a + bi \):
   
   (a) \(e^{-\pi i/4}\);
   (b) \(e^{1+\pi i}\);
   (c) \(e^{3i+i}\).

6. Find all complex numbers \( z \) such that \( z^4 = -1 \). Write the answer in both polar and cartesian coordinates. How many different solutions are there?

7. Find all complex numbers \( z \) such that \( z^5 = -2 - 2i \). (You can leave your answer in polar form.) How many different solutions are there?

8. Solve the equation \( z^2 + \sqrt{32}iz - 6i = 0 \).

9. Recall from class that for any two complex numbers \( z_1, z_2 \in \mathbb{C} \), we have the triangle inequality:
   
   \[ |z_1 + z_2| \leq |z_1| + |z_2| \]
   
   (a) Give an example when this inequality is strict; that is, when \( |z_1 + z_2| < |z_1| + |z_2| \).
   
   (b) When can equality occur?
   
   (c) Using the triangle inequality and a judicious choice of \( z_1 \) and \( z_2 \), prove the reverse triangle inequality:
   
   \[ |z_1 - z_2| \geq |z_1| - |z_2| \]

10. Write the function \( f(z) \) in the form \( u + iv \):
    
    (a) \(z + iz^2\);
    (b) \(1/z^2\);
    (c) \(\mathcal{Z}/z\).

11. Is the function \( \mathcal{Z}/z \) continuous at 0? Why or why not? Is the function \( \mathcal{Z}/z \) analytic where it is defined? Why or why not?

12. Compute the derivatives of the following analytic functions:
    
    (a) \(\frac{iz + 3}{z^2 - (2 + i)z + (4 - 3i)}\);
    (b) \(e^{z^2}\);
    (c) \(\frac{1}{e^z + e^{-z}}\).

13. Let \( f(z) \) be a complex function. Is it possible for both \( f(z) \) and \( f(z) \) to be analytic? (Hint: if they are both analytic, they both satisfy the Cauchy-Riemann equations.)