Name: Practice Midterm

1. (a) Are the vectors $\langle 1, 5, 3 \rangle$ and $\langle -3, 0, 2 \rangle$ perpendicular?
(b) Are the points $P = (1, -1, 1), Q = (2, 1, -1), R = (0, 3, -1), S = (-1, 0, 1)$ coplanar?
(c) Find the area of the triangle with vertices $A = (1, 0, 0), B = (0, 1, 0), C = (0, 0, 2)$.

2. (a) Find an equation for the plane through the points $P = (1, 0, 1), Q = (1, 2, 2), R = (3, 1, 2)$.
(b) Find the intersection of the line 
$$r(t) = (1, 1, 0) + t(3, 1, 1)$$
with the plane $x + y + z = 1$.

3. Consider the following two lines:
$$L_1: \quad x = \frac{y - 1}{2} = \frac{z - 1}{3}$$
$$L_2: \quad 6(x - 1) = 3y + 1 = z.$$
(a) Show that $L_1$ and $L_2$ are skew.

4. Consider the point $P = (0, 0, 1)$ and the plane $x + y + z = 0$.
(a) Find the distance between $P$ and the plane.
(b) Find the point $Q$ on the plane which is closest to $P$.

5. Consider the parametric equations
$$x(t) = |3 \cos(t)|$$
$$y(t) = \frac{\sin(t)}{2},$$
with $t \in [0, 2\pi]$.
(a) Identify the curve that the equations trace out, sketch the curve, and describe how the equations trace out the curve.
(b) Suppose we change the equations to
$$x(t) = |3 \cos(2t)| = |3 \cos(-2t)|$$
$$y(t) = \frac{-\sin(2t)}{2} = \frac{-\sin(-2t)}{2}.$$
How does this change the parametrization of the curve?
6. Let \( \vec{u}(t) = \langle e^{t^2}, 7, t^4 \rangle \), \( \vec{v}(t) = \langle \ln(t), \tan(t), 0 \rangle \) for \( t > 0 \).

(a) Compute \( \vec{u}'(t), \vec{u}''(t), \vec{v}'(t), \) and \( \vec{u}(t) \times \vec{v}(t) \).

(b) Find the equation for the tangent line to the curve given by the function \( \vec{u}(t) \) at the point \( P = (e^4, 7, 16) \).

7. Let a curve \( C \) be parametrized by the vector-valued function
\[
\vec{r}(t) = (\cos(t) + t \sin(t))\hat{i} + (\sin(t) - t \cos(t))\hat{j} + (\sqrt{\frac{3}{2}} t^2)\hat{k}.
\]

(a) Compute \( \vec{r}(\pi/2), \vec{r}'(t), \) and \( \vec{r}''(\pi/2) \).

(b) Find the equation for the tangent line to the curve at \( \pi/2 \).

(c) Compute the length of the curve from \( t = 0 \) to \( t = 2\pi \).