Name:

Calculus I Midterm #2

April 1, 2008

I. Limits: compute or state the limit does not exist. Explain your work. (5 points each)

(a) $\lim_{\theta \to 0} \frac{\sin(4\theta)}{\theta}$

(b) $\lim_{r \to 0^+} r^r$

(c) $\lim_{x\to\infty} x^{100}e^{-x}$

(d) $\lim_{n\to\infty} (1+\frac{13}{n})^n$ (Hint: use the substitution $m = \frac{n}{13}$.)

(e) $\lim_{x\to 0^+} \sin x \ln x$

II. Derivatives: find the derivative of the function. Make obvious simplifications to your answer, but do not worry about returning the most compact expression. (5 points each)

(a) $f(x) = \arctan(1 + x^2)$

(b) $f(x) = \frac{(x-1)^{1/5}}{(x^8-1)^{1/4}(x+4)^9}$

(c) $f(x) = \ln|\sin x|$

(d) $f(x) = \arcsin x$ (Hint: use implicit differentiation.)

(e) $f(x) = x^2 e^x \arcsin x$ (Use your answer in (d).)

III. Related Rates (6 points)

If the surface area of a cube is changing at a rate of 24 square centimeters per second when a side has length 1 centimeter, what is the rate of change (in cubic centimeters per second) of the volume of the cube at this instant? (The surface area of a cube with side length x is $6x^2$. The volume of a cube with side length x is x^3.)

IV. Linear approximation (6 points)

Find the linearization of $f(x) = x^6$ at $a = 2$. (Recall the linearization of $f(x)$ at $x = a$ is simply the linear function whose graph is the line tangent to the graph of f at a.) Then use it to estimate $(2.0001)^6$. Is this estimate greater than or less than the actual value?

V. Maximum/minimum values and Optimization (6 points each)

(a) Find the (global/absolute) maximum and minimum of $f(x) = \ln(x^2 + x + 1)$ on $[-1, 1]$ and where these extrema occur.

(b) If f is differentiable on all of $\mathbb{R}$ and $f'(c) = 0$, must f have a local extremum at c? (If so, cite the appropriate theorem; if not, give an example.)

VI. The Mean Value Theorem (5 points each)

Recall the Mean Value Theorem: if f is continuous on $[a, b]$ and differentiable on (a, b), then there exists $c \in (a, b)$ such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

(a) Show that $P(x) = x^5 + 3x + 7$ has at most one real root.

*(b) Let f and g be differentiable functions. Show that if $f(0) = g(0)$ and $f'(x) > g'(x)$ for all $x \geq 0$, then $f(x) > g(x)$ for all $x > 0$.

VII. Curve sketching (16 points total)

This problem concerns the function $f(x) = \frac{1}{1-x^2}$.

(a) What is the domain of f?

(b) Find any asymptotes (vertical, horizontal, slant).

(c) Compute $f'(x)$ and find the intervals on which f is increasing/decreasing. Find all local extrema and where they occur.

(d) Compute $f''(x)$ and find the intervals on which f is concave upward/downward.

(e) Sketch the graph of f. Label reasonably.