1. (20 pts) Compute $\int_{\gamma} (\sin(x)i + \cos(y)j + xk) \cdot dr$ where $r(t) = t^3i - t^2j + tk$ with $0 \leq t \leq 1$. 
(2) (20 pts)
(a) State Green’s Theorem. (You may state it in any of the forms which we covered in class.)
(b) Let \( \gamma \) be a simple closed curve in the plane. Using Green’s Theorem, show how \( \int_\gamma \mathbf{F} \cdot d\mathbf{r} \) is related to the area enclosed by \( \gamma \) when \( \mathbf{F} = -y\mathbf{d}x + x\mathbf{d}y \).

(3) (15 pts)
A particle starts at the point \((-2,0)\), moves along the x-axis to \((2,0)\), and then along the semicircle \( y = \sqrt{4 - x^2} \) to the starting point. Use Green’s Theorem to find the work done on this particle by the force field \( \mathbf{F}(x,y) = x\mathbf{i} + (x^3 + 3xy^2)\mathbf{j} \).
(4) (20 pts)
Let $\mathbf{F} = (y + e^{\sin(x)}, y^2)$ and let $\Delta$ be the triangle with vertices $(0,0)$, $(0,2)$, and $(1,0)$.
(a) (10 pts) Compute $\int_{\partial \Delta} \mathbf{F} \cdot d\mathbf{r}$. (Don’t forget the orientation convention for boundaries!)

(b) (10 pts) Let $\gamma$ denote the oriented curve consisting of the horizontal line from $(0,0)$ to $(1,0)$, followed by the straight line from $(1,0)$ to $(0,2)$. Compute $\int_{\gamma} \mathbf{F} \cdot d\mathbf{r}$. 
(5) (25 pts)
Consider the vector field \( \mathbf{F} = (x + y^2)\mathbf{i} + (2xy + y^2)\mathbf{j} \).
(a) Find a function \( f(x, y) \) for which \( \nabla f = \mathbf{F} \).

(b) Without using your calculation of \( f \), give another justification that \( \mathbf{F} \) is a gradient vector field.

(c) Using any method you wish, compute \( \int_C \mathbf{F} \cdot d\mathbf{r} \), where \( \mathbf{r}(t) = (e^{\cos(\pi t/2)} - \sqrt{1 - t}e^{t^3})\mathbf{i} + (t^5)\mathbf{j} \) for \( t \in [0, 1] \).

(6) Extra Credit. Consider the vector field \( \mathbf{F}(x, y, z) = -\frac{y}{x^2 + y^2}\mathbf{i} + \frac{x}{x^2 + y^2}\mathbf{j} + \mathbf{k} \), whose domain is \( \mathbb{R}^3 \) minus the \( z \)-axis.
(a) Calculate the integral of \( \mathbf{F} \) around the circle given by \( x^2 + y^2 = 1 \) and \( z = 0 \).
(b) Why is this example interesting in relation to what has been discussed in the class thus far?