Calc 1 Spring 2005 Sample Final Exam

Please write full solutions, not just answers. Unless otherwise marked, answers without justification will get little or no partial credit. Cross out anything the grader should ignore and circle or box the final answer. Do NOT round answers. Calculators are not allowed.

(1) (25 points) Calculate \( f'(x) \) of the following functions:
   (a) \( f(x) = \frac{x^2}{\cos(x)} \)
   (b) \( f(x) = \ln(x^5 + 7) \)
   (c) \( f(x) = \int_{-3}^{x} e^{3t-4} \sin(t) \) dt
   (d) \( f(x) = 2^x + \frac{1}{x} \)

(2) (30 points) Let \( f(x) = |1 + \frac{1}{x}| \).
   (a) Sketch the graph of \( f \) and \( f' \). Identify the asymptotes.
   (b) Find all values of \( x \) for which \( f \) is not continuous.
   (c) Find all values of \( x \) for which \( f \) is not differentiable.
   (d) What is the area under \( f(x) \) between 4 and 7?

(3) (50 points) Compute the following integrals:
   (a) \( \int_{0}^{x} t^3 dt \)
   (b) \( \int_{-\pi}^{\pi} (\sin(x) - \cos(x)) dx \)
   (c) \( \int_{-\pi}^{\pi} \sin(x) \cos(x) dx \)
   (d) \( \int_{a}^{b} x^2 \sqrt{x^3 + 1} \) dx
   (e) \( \int_{1}^{4} \sqrt{x} \) dx
   (f) \( \int_{0}^{e} e^{3x} dx \)
   (g) \( \int_{1}^{e} \frac{1}{t} dt \)

(4) (25 points) Using the definition of integral, calculate \( \int_{-1}^{3} 2dx \).

(5) (25 points) A closed rectangular container with square base is to have a volume of 2250 cubic inches. The material for the top and bottom of the container will cost $2 per square inch and the material for the sides will cost $3 per square inch. Find the dimensions of the container of least cost.

(6) (20 points) Calculate the area of the bounded region between the curves \( f(x) = x^4 \) and \( g(x) = x \).

(7) (25 points) A car moving at 60 mi/h along a straight road passes under a weather balloon rising vertically at 30 mi/h. If the balloon is 1 mi up when the car is directly beneath it, how fast is the distance between the car and the balloon increasing 4 minutes later?

(8) (a) Write down the definition of \( f'(x) \).
   (b) Using the definition of derivative, show that \( \frac{d}{dx}(\sin(x)) = \cos(x) \).
   Hint: you are allowed to assume the fact that \( \lim_{h \to 0} \frac{\sin(h)}{h} = 1 \)
   and that \( \lim_{h \to 0} \frac{\cos(h) - 1}{h} = 0 \).