# FINAL EXAM

MAT 415H1F/1200HF Fall 2020

**DUE: Thursday, December 17th at 6pm**

Turn into Crowdmark.

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**You are allowed to consult any of the references listed on the course website during the exam, as well as any lecture notes and previous homework. DO NOT DISCUSS THE EXAM WITH ANYONE OTHER THAN THE INSTRUCTOR UNTIL AFTER THE DEADLINE.**

Feel free to use any previous homework problems and any proven statements from lecture without proof. Carefully cite what you use!

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(1a) Compute the class number of $\mathbb{Q}(\sqrt{-35})$, and prove your answer is correct.

(1b) Find all integer solutions to $a^3 = b^2 + 35$. Justify your answer.

(2) Prove that all primes $p \equiv 1 \bmod 3$ can be written as $a^2 + ab + b^2$.

(3a) Compute the ring of integers and the discriminant of $K = \mathbb{Q}(\sqrt{2}, \sqrt{-3})$. Justify your answer.

(3b) Describe the prime factorizations of (6) and (19) in $\mathcal{O}_K$. Justify your answer.

(3c) Find generators for $\mathcal{O}_K^\times$. Justify your answer.

(4) Find a quadratic field whose class number is divisible by 1024. (Hint: $1024 = 2^{10}$). Justify your answer.

(5) Exercise 4.46 on Pg. 106-7 of [B]

(6) Exercise 5.10 on Pg. 118 of [B]