

Business Calculus I (Math 221) Quiz

Date: December 10, 2014

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Justify answers and show all work for full credit.

NAME: Key

Problem 1. Evaluate

(a) $\int_1^4 5x^{3/2} dx = 5 \cdot \frac{2}{5} x^{5/2} \Big|_1^4$

$$= 2 [4^{5/2} - 1^{5/2}] = 2(32 - 1) = 62$$

(b) $\int_0^1 \frac{x^3}{3x^4 + 5} dx$ $u = 3x^4 + 5$ $x=0 \Rightarrow u=5$
 $du = 12x^3 dx$ $x=1 \Rightarrow u=8$

$$= \frac{1}{12} \int_5^8 \frac{1}{u} du = \frac{1}{12} \ln|u| \Big|_5^8 = \frac{1}{12} (\ln 8 - \ln 5) \approx 0.0392$$

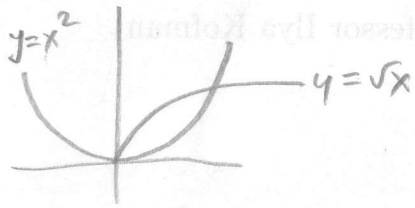
(c) $\int_{-1}^2 (7x+3)^{-2} dx$ $u = 7x+3$ $x=-1 \Rightarrow u=-4$
 $du = 7dx$ $x=2 \Rightarrow u=17$

$$= \frac{1}{7} \int_{-4}^{17} u^{-2} du = -\frac{1}{7} u^{-1} \Big|_{-4}^{17} = \frac{1}{7} \left(-\frac{1}{4} - \frac{1}{17} \right)$$

Note: This integral does not converge! $= \frac{3}{68}$

I mistakenly put an improper integral on the quiz - note that $(7x+3)^{-2}$ has an asymptote at $x = -3/7$.

Problem 2. Find the area enclosed by the graphs $y = x^2$ and $y = \sqrt{x}$.



$$\text{Set } x^2 = \sqrt{x} \Rightarrow x^4 - x = 0$$

$$x(x^3 - 1) = 0$$

$$x = 0 \text{ or } x = 1$$

$$A = \int_0^1 \sqrt{x} - x^2 dx = \left[\frac{2}{3} x^{3/2} - \frac{1}{3} x^3 \right]_0^1$$

$$= \frac{2}{3}(1-0) - \frac{1}{3}(1-0) = \frac{1}{3}$$

Problem 3. An income stream has annual flow rate $f(t) = 4000 e^{0.05t}$ for 10 years.

(a) Find the total income over the next 10 years.

(b) Find the present value using an interest rate of 3% (APR).

$$\text{a) Total} = \int_0^{10} 4000 e^{0.05t} dt$$

$$= \frac{4000}{0.05} e^{0.05t} \Big|_0^{10}$$

$$= 80,000 (e^{0.5} - e^0) = \$51,897.70$$

$$\text{b) PV} = \int_0^{10} (4000 e^{0.05t}) e^{-0.03t} dt$$

$$= \int_0^{10} 4000 e^{0.02t} dt$$

$$= \frac{4000}{0.02} e^{0.02t} \Big|_0^{10} = 200,000 (e^{0.2} - 1)$$

$$= \$44,280.55$$