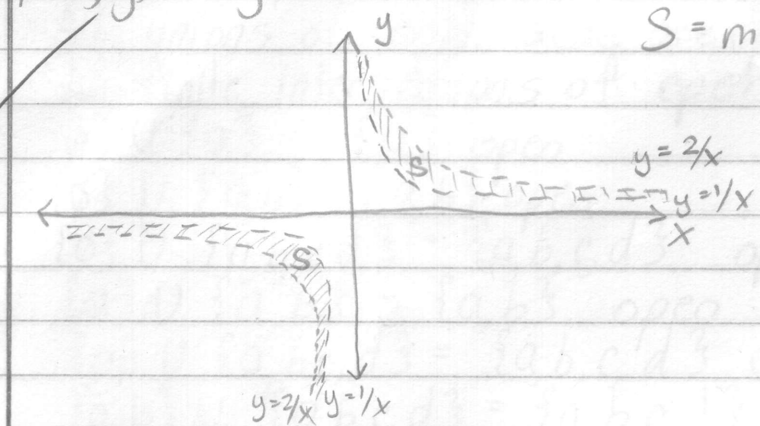
3.4 Let $f: S \to T$ between two topological spaces.
(→) If f is continuous and $C \subset T$ is closed, $T-C$ is open and $f^{-1}(T-C)$ is open. Therefore, $f^{-1}(T-C) = S - f^{-1}(C)$ is open and $f^{-1}(C)$ is closed.
(←) *If $C \subset T$ is closed, $f^{-1}(C)$ is closed. Therefore, $S - f^{-1}(C) = f^{-1}(T-C)$ is open when $T-C$ is open and f is continuous. *Given open U, let $C = U^c = T - U$

3.6 Let $m: R^2 \to R$ be the multiplication function $m(x,y) = xy$



$S = m^{-1}((1,2))$
$S = \{(x,y) \mid 1 < xy < 2\}$ open interval
This is open because there is an open neighborhood around every point in the preimage.

3.8 Let $f: \mathbb{R} \to \mathbb{Z}$ be the function $f(x) = n$ for $n \in \mathbb{Z}$ and $n \le x < n+1$. If f is continuous, $f^{-1}(S)$ is open for all $S \subset \mathbb{Z}$ which are open. However, every integer $n \in \mathbb{Z}$ is an open set and the preimage of $n = f^{-1}(n) = [n, n+1)$ which is not open. Therefore, f is not continuous.

3.9