Mth 233 Quiz 1 October 24, 2016 Solutions

① $\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial s} = -e^y\cdot 2 + (ye^y + e^y - xe^y)\cdot t$

$\frac{\partial f}{\partial s}(5,3) = -2e^{15} + (15e^{15} + e^{15} - 7e^{15})\cdot 3 = \underline{\underline{25e^{15}}}$

---

② $f(x,y) = 6xy - x^3 - y^3 \Rightarrow \nabla f = (6y - 3x^2, 6x - 3y^2) \stackrel{\text{set}}{=} 0$

$\left.\begin{array}{l} i)\ 6y - 3x^2 = 0 \Rightarrow 2y = x^2 \\ ii)\ 6x - 3y^2 = 0 \Rightarrow 2x = y^2 \end{array}\right\}$ $2x = \left(\frac{x^2}{2}\right)^2 = \frac{x^4}{4} \Rightarrow$ $x=0$ or $x^3 = 8$ ie. $x = 2$; $(0,0)$, $(2,2)$

CP: $(0,0)$ and $(2,2)$

$\left.\begin{array}{ll} f_{xx} = -6x & f_{xy} = 6 \\ f_{yx} = 6 & f_{yy} = -6y \end{array}\right. \Rightarrow D = 36xy - 36$

$D(0,0) = -36 < 0$ saddle at $(0,0)$

$D(2,2) = 108 > 0$, $f_{xx}(2,2) = -12 < 0$ max at $(2,2)$

---

③ $f(x,y) = 4x^2 + 9y^2 \Rightarrow \nabla f = (8x, 18y) = \lambda\nabla g = \lambda(2,3)$

$g(x,y) = 2x + 3y - 6$

$\left.\begin{array}{ll} i)\ 8x = 2\lambda & 4x = \lambda \\ ii)\ 18y = 3\lambda & 6y = \lambda \end{array}\right\}$ $2x + 3y = 6$; $2\left(\frac{\lambda}{4}\right) + 3\left(\frac{\lambda}{6}\right) = 6 \Rightarrow \underline{\lambda = 6}$

a) CP when $\lambda = 6$ is $(x,y) = \left(\frac{3}{2}, 1\right)$, so $f\left(\frac{3}{2},1\right) = 18$ is extreme value

b) This extremum is a <u>minimum</u> because $\lim_{x\to\pm\infty} f(x,y) = +\infty$ for (x,y) on $2x + 3y = 6$.

---

④ $f(x,y,z) = 2x + 6y + 10z \Rightarrow \nabla f = (2,6,10) = \lambda\nabla g = \lambda(2x, 2y, 2z)$

$g(x,y,z) = x^2 + y^2 + z^2 - 35$

(i) $2 = 2\lambda x$, $1 = \lambda x$ (ii) $6 = 2\lambda y$, $3 = \lambda y$ (iii) $10 = 2\lambda z$, $5 = \lambda z$ $\Rightarrow$

$x^2 + y^2 + z^2 = 35$

$\left(\frac{1}{\lambda}\right)^2 + \left(\frac{3}{\lambda}\right)^2 + \left(\frac{5}{\lambda}\right)^2 = 35$

$1 + 9 + 25 = 35\lambda^2 \Rightarrow \underline{\lambda = \pm 1}$

CP: $(1,3,5)$ and $(-1,-3,-5)$ first one is max, second one is min by evaluating $f(1,3,5) = 70$ $f(-1,-3,-5) = -70$

---

⑤ a) $\nabla f = (6x, 4y - 4) \stackrel{\text{set}}{=} 0 \Rightarrow$ CP: $(0,1)$

b) $\nabla f = (6x, 4y-4) = \lambda\nabla g = \lambda(2x, 2y) \Rightarrow$ (i) $6x = 2\lambda x$ (ii) $4y - 4 = 2\lambda y$

$x = 0$ or $\lambda = 3$ $\quad 2(y-1) = \lambda y$

If $x = 0$, $y = \pm 3$. If $\lambda = 3$, $y = -2 \Rightarrow x = \pm\sqrt{5}$

So CP: $(0,3)$, $(0,-3)$, $(\sqrt{5}, -2)$, $(-\sqrt{5}, -2)$

c) $f(0,1) = -2$, $f(0,3) = 6$, $f(0,-3) = 30$, $f(\pm\sqrt{5}, -2) = 31 \Rightarrow$ $\left\{\begin{array}{l}\text{Absolute min at } (0,1) \\ \text{Absolute max at } (\pm\sqrt{5}, -2)\end{array}\right.$