

# Quiz 1 Solutions 10/25/2010

①  $f(x, y) = 2x^2 + y^2 - 6y$ ,  $x^2 + 2y^2 \leq 25$

b)  $\nabla f = (4x, 2y - 6) = \lambda (2x, 4y) = \lambda \nabla g$

$4x = \lambda \cdot 2x$  and  $2y - 6 = \lambda \cdot 4y$  (both eq. must hold)

$2x - \lambda x = 0$

$x(2 - \lambda) = 0$

$x = 0$  or  $\lambda = 2$

$y - 3 = 2 \cdot \lambda y$

If  $\lambda = 2$  then  $y = -1$

If  $x = 0$ ,  $x^2 + 2y^2 = 25 \Rightarrow (0, \pm \frac{5}{\sqrt{2}})$

If  $y = -1$ ,  $x^2 + 2y^2 = 25 \Rightarrow (\pm \sqrt{23}, -1)$

a) For  $x^2 + 2y^2 < 25$ , set  $\nabla f = 0 \Rightarrow x = 0$  and  $y = 3$   
(0, 3)

c) Evaluate  $f(x, y)$ : Max  $f(\pm \sqrt{23}, -1) = 53$   
Min  $f(0, 3) = -9$   
on all five pts.

②  $f(x, y) = 6xy - x^3 - y^3 \Rightarrow \nabla f = (6y - 3x^2, 6x - 3y^2) \stackrel{\text{set}}{=} 0$

$6y - 3x^2 = 0 \Rightarrow 2y = x^2$ ,  $6x - 3y^2 = 0 \Rightarrow 2x = y^2$

$\Rightarrow 2x = (\frac{x^2}{2})^2 = \frac{x^4}{4} \Rightarrow x(x^3 - 8) = 0 \Rightarrow x = 0$  or  $x = 2$

$x = 0 \Rightarrow y = 0$ ,  $x = 2 \Rightarrow y = 2$  So we set (0, 0) and (2, 2)

$D = 36xy - 36 < 0$  (saddle) at (0, 0)

$> 0$ ,  $f_{xx} = -6x < 0$  at (2, 2)  $\Rightarrow$  max