Math 233 Exam 1 9/29/2010 Solutions

① $\ell(t) = (2+3t, -t, 4+t)$. Let $Q = \ell(0) = (2,0,4)$. Let $\vec{v} = (3,-1,1)$

So $\ell(t) = Q + t\vec{v}$. Hence, plane contains $\vec{v}$ and $\vec{PQ} = (2,-2,5)$.

Let $\vec{n} = \vec{v} \times \vec{PQ} = \begin{vmatrix} i & j & k \\ 3 & -1 & 1 \\ 2 & -2 & 5 \end{vmatrix} = (-3,-13,-4) \Rightarrow (3,13,4)\cdot(\vec{x}-\vec{P}) = 0$

$(3,13,4)\cdot(x, y-2, z+1) = 0 \Rightarrow \underline{3x+13y+4z = 22}$

② a) $\vec{r}\,'(t) = (3, 4\cos 4t, -4\sin 4t) \Rightarrow v(t) = \|\vec{r}\,'(t)\| = \sqrt{9+16} = 5$

b) $T(t) = \dfrac{\vec{r}\,'(t)}{\|\vec{r}\,'(t)\|} = \frac{1}{5}(3, 4\cos 4t, -4\sin 4t)$

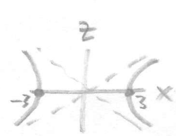
c) $T'(t) = \frac{1}{5}(0, -16\sin 4t, -16\cos 4t) \Rightarrow N(t) = \dfrac{T'(t)}{\|T'(t)\|} = (0, -\sin 4t, -\cos 4t)$

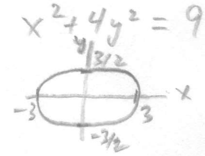
Verify: $T\cdot N = 0 - \frac{4}{5}\sin 4t\cos 4t + \frac{4}{5}\sin 4t \cos 4t = 0 \Rightarrow T \perp N$

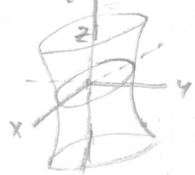
d) $\kappa = \dfrac{\|T'(t)\|}{\|\vec{r}\,'(t)\|} = \dfrac{16/5}{5} = \dfrac{16}{25}$ ($\kappa(t)$ is constant for this $\vec{r}(t)$.)

e) $L = \int_{\pi}^{2\pi} \|\vec{r}\,'(t)\|\,dt = \int_{\pi}^{2\pi} 5\,dt = 5\pi$

③ a) If $x=0$, $4y^2 - z^2 = 9$ 

If $y=0$, $x^2 - z^2 = 9$ 

If $z=0$, $x^2 + 4y^2 = 9$ 

So S is a 1-sheet hyperboloid: 

b) Let $z = f(x,y) = \sqrt{x^2+4y^2-9}$

$f_x = \frac{1}{2}(x^2+4y^2-9)^{-1/2}(2x) \Rightarrow f_x(-3,2) = \dfrac{-6}{2\sqrt{16}} = -\dfrac{3}{4}$

$f_y = \frac{1}{2}(x^2+4y^2-9)^{-1/2}(8y) \Rightarrow f_y(-3,2) = \dfrac{16}{2\sqrt{16}} = 2$

Tan plane $z = L(x,y) = 4 + (-\frac{3}{4})(x+3) + (2)(y-2) = -\frac{3}{4}x + 2y - \frac{9}{4}$

$\Rightarrow 3x - 8y + 4z = -9$

④ a) If $x=0$, $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^3y}{2x^4+y^4} = \lim\limits_{y\to 0} \dfrac{0}{0+y^4} = 0$ (Similarly if $y=0$)

If $x=y$, $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^3y}{2x^4+y^4} = \lim\limits_{x\to 0} \dfrac{x^4}{3x^4} = \dfrac{1}{3}$ $\Big\}\Rightarrow$ limit DNE.

b) $h_x = x\cos(x+2y) + \sin(x+2y) \Rightarrow h_{xy} = -2x\sin(x+2y) + 2\cos(x+2y)$

$h_y = 2x\cos(x+2y) \Rightarrow h_{yx} = -2x\sin(x+2y) + 2\cos(x+2y)$ $\Big\}$ equal

⑤ If R is the radius of earth, then $\|\vec{r}(t)\| = R$. Thus $\vec{r}\cdot\vec{r} = \|\vec{r}\|^2 = R^2$.

$\Rightarrow \frac{d}{dt}(\vec{r}\cdot\vec{r}) = 0 \Rightarrow \vec{r}\cdot\vec{r}\,' + \vec{r}\,'\cdot\vec{r} = 0 \Rightarrow 2\vec{r}\cdot\vec{r}\,' = 0 \Rightarrow \vec{r}\cdot\vec{r}\,' = 0 \Rightarrow \underline{\underline{\vec{r}\perp\vec{r}\,'!}}$