Sample problems for Exam 1 for Math 232

Fall 2014, Prof. Ilya Kofman

- This sample exam has more questions than the actual exam will have.
- Study sections 5.6–5.7, 6.1–6.4, 7.1–7.3, 7.5, and Quizes.

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1. Evaluate the following integrals.

   (a) $\int x\sin(2x^2-3)\,dx$

   (b) $\int_0^1 e^{5-5y}\,dy$

   (c) $\int \frac{1}{\sqrt{3t+2}}\,dt$

   (d) $\int \frac{s^2}{(s^3+5)^5}\,ds$

   (e) $\int_1^2 \frac{3}{t^2+4t+5}\,dt$

   (f) $\int \frac{1}{\sqrt{16-6y-y^2}}\,dy$

   (g) $\int \frac{3x+4}{x^2+4}\,dx$

   (h) $\int \frac{3}{as+b}\,ds$

2. Find the equation of tangent to the curve $y = \arcsin(x^3-1)$ at the point $(1,0)$.

3. Sketch the region enclosed by the curves $y = x^2-2$ and $y = 6-x^2$, and find its area.

4. Sketch the region enclosed by the curves $y = \sin 2x$ and $y = \cos x$ and the lines $x = 0$ and $x = \pi/2$, and find its area.

5. Find the volume of the solid obtained by revolving the region bounded by the curves $y = x^2$ and $x = y^2$ about the line $x = -1$ using the washer method. Do the same but now with the shell method.

6. Set up the integrals to find the volume of the solid obtained by revolving the region bounded between the curves $y = x^2-1$ and the line $y = x+5$ about the following axes:

   (a) $y = 10$ (Use washer method)

   (b) $x = 3$ (Use shell method)

7. Find the volume of the given solid obtained by rotating the region bounded by given curves about the specified axis.

   (a) $y = x^2,\ y = 0, x = 2$, rotated about x–axis.
   (b) $y = x^3,\ y = 2x - x^2$ in the first quadrant, rotated about x–axis.
   (c) $y = x^3,\ y = 2x - x^2$ in the first quadrant, rotated about y–axis.

8. Find the volume of a solid whose base is a region between the curve $y = 2\sin x$ and the interval $0 \le x \le \pi$ on the x–axis, and the cross sections perpendicular to the x–axis are squares with one side placed between the x–axis and the curve.

9. Evaluate the following integrals.

   (i) Substitution:
   (a) $\int e^{4x+3}\, dx$ (b) $\displaystyle\int \frac{1}{ax+b}\, dx$ (c) $\displaystyle\int_0^1 \frac{3t}{(t^2+4)^5}\, dt$
   (d) $\int \sin(ay+b)\, dy$ (e) $\displaystyle\int_1^{e^2} \frac{4+\ln x}{x}\, dx$

   (ii) Integration by parts:
   (a) $\int xe^{5x}\, dx$ (b) $\int s^2e^{3s}\, ds$ (c) $\int_1^2 \ln x\, dx$
   (d) $\int t^3 \ln t\, dt$ (e) $\int \sin x\, e^x\, dx$ (f) $\int x^2 \sin x\, dx$

   (iii) Trigonometric Integrals:
   (a) $\int \cos^2 x\, dx$ (b) $\int_0^\pi \sin^2 x\, dx$ (c) $\int \tan y\, dy$
   (d) $\int \cos^2 x\ \sin^3 x\, dx$ (e) $\int \sec t\, dt$

   (iv) Trigonometric substitution: (Leave final answer in terms of x)
   (a) $\int \frac{dx}{\sqrt{9-x^2}}$ (b) $\int \frac{dx}{x\sqrt{x^2-9}}$ (c) $\int \frac{dx}{x^2\sqrt{x^2+9}}$

   (v) Partial Fractions:
   (a) $\displaystyle\int \frac{dx}{x^2-6x-16}$ (b) $\displaystyle\int \frac{7t+1}{t^2+t-6}\, dt$ (c) $\displaystyle\int \frac{dy}{y^2-a^2}$

   (vi) Partial Fractions with Long Division:
   (a) $\displaystyle\int \frac{9t+1}{3t+4}\, dt$ (b) $\displaystyle\int \frac{x^3+4x-3}{x^2+4}\, dx$ (c) $\displaystyle\int \frac{3y^2+2}{y^2+4}\, dy$