Calculus II (Math 232) Quiz

October 29, 2014 — Professor Ilya Kofman

Justify answers and show all work for full credit.

NAME: Key

1. Use the shell method to calculate the volume of the infinite solid obtained by rotating about the y–axis the region under $y = \dfrac{1}{(x^2+25)^2}$ for $0 \le x < \infty$.

2. Calculate the volume of the infinite solid obtained by rotating about the x–axis the region under $y = \dfrac{1}{\sqrt{x^2+9}}$ for $0 \le x < \infty$.

3. Use the Comparison Test to determine whether the following integral converges or diverges:
$$\int_0^\infty \frac{1}{\sqrt{x^2+9}}\,dx$$

4. Let $f(x) = \sqrt{2x+1}$. Compute the Taylor polynomial $T_3(x)$ centered at $a = 1$ for $f(x)$.

① $V = 2\pi\int rh\,dx = 2\pi\int_0^\infty (x)\frac{1}{(x^2+25)^2}\,dx = \lim\limits_{R\to\infty} 2\pi\int_0^R \frac{x}{(x^2+25)^2}\,dx$ let $u = x^2+25$, $du = 2x\,dx$

$= \lim\limits_{R\to\infty} \pi\int_{25}^{R^2+25} u^{-2}\,du = \lim\limits_{R\to\infty} -\frac{\pi}{u}\Big|_{25}^{R^2+25} = \lim\limits_{R\to\infty} \pi\left(\frac{1}{25} - \frac{1}{R^2+25}\right) = \pi/25$

② $V = \pi\int_0^\infty r^2\,dx = \pi\int_0^\infty \left(\frac{1}{\sqrt{x^2+9}}\right)^2 dx = \pi\int_0^\infty \frac{1}{x^2+9}\,dx = \lim\limits_{R\to\infty} \pi\int_0^R \frac{1}{x^2+9}\,dx$

$= \lim\limits_{R\to\infty} \pi\cdot\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)\Big|_0^R = \lim\limits_{R\to\infty} \frac{\pi}{3}\left(\tan^{-1}\left(\frac{R}{3}\right) - \tan^{-1}(0)\right) = \frac{\pi}{3}\cdot\frac{\pi}{2} = \frac{\pi^2}{6}$

③ $\int_0^\infty \frac{1}{\sqrt{x^2+9}} \ge \int_1^\infty \frac{1}{\sqrt{x^2+9}} \ge \int_1^\infty \frac{1}{\sqrt{10x^2}} = \frac{1}{\sqrt{10}}\int_1^\infty \frac{1}{x}\,dx = \infty$ (diverges, p-integral $p=1$)

$x^2+9 \le 10x^2$ if $x \ge 1$

<u>diverges</u>

④ $f(x) = \sqrt{2x+1}$ $\quad f'(x) = \cancel{\tfrac{1}{2}}(2x+1)^{-1/2}\cancel{(2)}$ $\quad f''(x) = -\cancel{\tfrac{1}{2}}(2x+1)^{-3/2}\cancel{(2)}$ $\quad f'''(x) = \frac{3}{\cancel{2}}(2x+1)^{-5/2}\cancel{(2)}$

$f(1) = \sqrt{3}$ $\quad f'(1) = \frac{1}{\sqrt{3}}$ $\quad f''(1) = \frac{-1}{3\sqrt{3}}$ $\quad f'''(1) = \frac{3}{9\sqrt{3}} = \frac{1}{3\sqrt{3}}$

$T_3(x) = \sqrt{3} + \frac{1}{\sqrt{3}}(x-1) - \frac{1}{2}\cdot\frac{1}{3\sqrt{3}}(x-1)^2 + \frac{1}{6}\cdot\frac{1}{3\sqrt{3}}(x-1)^3$