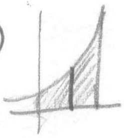
Mth 232 Exam 1 Solutions 10/15/2014

① $V=\int_{-R}^{R}\pi r^2\,dx = \int_{-R}^{R}\pi(R^2-x^2)\,dx = \pi\left[R^2x-\frac{1}{3}x^3\right|_{-R}^{R} = \frac{4}{3}\pi R^3$

②  $V=\int_0^1 2\pi x h\,dx = 2\pi\int_0^1 xe^x\,dx$ $\quad u=x \quad dv=e^x dx$, $\; du=dx \quad v=e^x$ $\quad = 2\pi\left[xe^x-\int e^x dx\right|_0^1$

$= 2\pi\left(xe^x-e^x\right|_0^1 = 2\pi\left(0-(-1)\right) = 2\pi$

③ (Shell method) $V=\int_{-2}^{1} 2\pi r h\,dy = 2\pi\int_{-2}^{1}(1-y)\left((2-y)-y^2\right)dy$

$= 2\pi\int_{-2}^{1} y^3-3y+2\,dy = 2\pi\left[\frac{1}{4}y^4-\frac{3}{2}y^2+2y\right|_{-2}^{1} = \frac{27\pi}{2}$

④ (Washer method) $V=\int_{-2}^{1}\pi(R^2-r^2)\,dy = \pi\int_{-2}^{1}(4-y^2)^2-(4-(2-y))^2\,dy$

$= \pi\int_{-2}^{1}(16-8y^2+y^4)-(2+y)^2 = \pi\int_{-2}^{1} y^4-9y^2-4y+12\,dy$

⑤ $u=\ln x \Rightarrow \int u^{-1/3}\,du = \frac{3}{2}u^{2/3}+C = \frac{3}{2}(\ln x)^{2/3}+C$

$du=\frac{1}{x}\,dx$

⑥ $\cos^2(18x)=\frac{1}{2}(1+\cos(36x)) \Rightarrow \frac{1}{2}\int 1+\cos(36x)\,dx = \frac{1}{2}\left(x+\frac{1}{36}\sin(36x)\right)$

$= \frac{1}{2}x+\frac{1}{72}\sin(36x)+C$

⑦ $u=x^2 \quad dv=\sin(3x)\,dx \Rightarrow -\frac{x^2}{3}\cos(3x)-\int -\frac{2}{3}x\cos(3x)\,dx$ $\quad u=x \quad dv=\cos(3x)\,dx$, $\; du=dx \quad v=\frac{1}{3}\sin(3x)$

$du=2x \quad v=-\frac{1}{3}\cos(3x)$

$= -\frac{x^2}{3}\cos(3x)+\frac{2}{3}\left[\frac{1}{3}x\sin(3x)-\frac{1}{3}\int\sin(3x)\,dx\right]$

$= -\frac{x^2}{3}\cos(3x)+\frac{2}{9}x\sin(3x)+\frac{2}{27}\cos(3x)+C$

⑧ $3x=\sin\theta \Rightarrow \frac{1}{3}\int\sqrt{1-\sin^2\theta}\,\cos\theta\,d\theta = \frac{1}{3}\int\cos^2\theta\,d\theta = \frac{1}{3}\left(\frac{1}{2}\theta+\frac{1}{4}\sin(2\theta)\right)+C$

$3dx=\cos\theta\,d\theta$

$= \frac{1}{6}\sin^{-1}(3x)+\frac{1}{12}\cdot 2\underbrace{\sin\theta}_{3x}\underbrace{\cos\theta}_{\sqrt{1-9x^2}} = \frac{1}{6}\sin^{-1}(3x)+\frac{1}{2}x\sqrt{1-9x^2}+C$

⑨ $u=2x \quad dv=e^{6x}dx \Rightarrow \left[\frac{2}{6}xe^{6x}-\int\frac{2}{6}e^{6x}dx\right|_0^3 = \frac{1}{3}\left[xe^{6x}-\frac{1}{6}e^{6x}\right|_0^3 = \frac{17}{18}e^{18}+\frac{1}{18}$

$du=2dx \quad v=\frac{1}{6}e^{6x}$

~~people, on the other hand, he lets loose with a tirade of negative generalizations, but when~~

(10) $\dfrac{x^2+8x-15}{x^2(x-5)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x-5}$

$A(x)(x-5) + B(x-5) + C(x^2) = x^2+8x-15$

$$\left.\begin{array}{l} x=0 \Rightarrow -5B=-15 \Rightarrow B=3 \\ x=5 \Rightarrow 25C = 50 \Rightarrow C=2 \\ x^2 \text{ term} \Rightarrow A+C=1 \Rightarrow A=-1 \end{array}\right\} \begin{array}{l} \int \frac{-1}{x} + \frac{3}{x^2} + \frac{2}{x-5} = \\ -\ln|x| - \frac{3}{x} + 2\ln|x-5| + C \end{array}$$

(11) $\int \cos^3(4x)\,dx = \int \cos^2(4x)\cos(4x)\,dx = \int (1-\sin^2(4x))\cos(4x)\,dx$

$u=\sin(4x)$
$du = 4\cos(4x)$

$= \frac{1}{4}\int 1-u^2\,du = \frac{1}{4}\left(u - \frac{1}{3}u^3\right) + C = \underline{\frac{1}{4}\left(\sin(4x) - \frac{1}{3}\sin^3(4x)\right) + C}$

(12) $\int \dfrac{2x(x^2+16)+3}{x^2+16} = \int 2x + \dfrac{3}{x^2+16}\,dx = x^2 + \frac{3}{4}\tan^{-1}\left(\frac{x}{4}\right) + C$

using reduction formula, $\frac{1}{4}\left(\frac{1}{3}\cos^2 u \sin u + \frac{2}{3}\sin u\right)$

$= \underline{\frac{1}{12}\cos^2(4x)\sin(4x) + \frac{1}{6}\sin(4x) + C}$