MTH 231 Exam 3 11/23/2009 Solutions

① Let $f(x) = e^{\sin(x)}$, $a = \pi$, $\Delta x = 0.2$

$f(a+\Delta x) \simeq f(a) + f'(a)\Delta x$

$f'(x) = (e^{\sin(x)})(\cos x) \Rightarrow f'(\pi) = e^{\sin \pi} \cdot \cos \pi = e^{0} \cdot (-1) = -1$

$f(\pi + 0.2) \simeq e^{\sin \pi} + (-1)(0.2) = e^{0} - 0.2 = 1 - 0.2 = \underline{\underline{0.8}}$

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② $h(x) = \sin x - \frac{x}{2} \Rightarrow h'(x) = \cos x - \frac{1}{2} \overset{\text{set}}{=} 0$

CP: $\cos x = \frac{1}{2}$ for $0 \le x \le \pi \Rightarrow x = \frac{\pi}{3}$ (CP) $\quad h(\frac{\pi}{3}) = \sin\frac{\pi}{3} - \frac{\pi}{6} = \frac{\sqrt{3}}{2} - \frac{\pi}{6} \simeq 0.9 - 0.5 > 0$

Endpts: $h(0) = \sin 0 - 0 = 0$

$h(\pi) = \sin \pi - \frac{\pi}{2} = -\frac{\pi}{2}$ (Min value of h)

$\Rightarrow$ Max at $x = \frac{\pi}{3}$, Min at $x = \pi$

Max value $= \frac{\sqrt{3}}{2} - \frac{\pi}{6}$ Min value $= -\frac{\pi}{2}$

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③ $g(x) = \frac{3}{5}x^5 - 2x^3 - 2 \Rightarrow g'(x) = 3x^4 - 6x^2 \Rightarrow g''(x) = 12x^3 - 12x$

$= 3x^2(x^2-2) \qquad = 12x(x^2-1)$

$= 3x^2(x-\sqrt{2})(x+\sqrt{2}) \qquad = 12x(x-1)(x+1)$

a) CP: $g'(x) = 0 \Rightarrow x = 0, \sqrt{2}, -\sqrt{2}$

First Derivative Test:

		$-\sqrt{2}$		0		$\sqrt{2}$	
$3x^2$	+		+		+		+
$x-\sqrt{2}$	−		−		−		+
$x+\sqrt{2}$	−		+		+		+
$g'(x)$	+		−		−		+
$g(x)$	inc	max	dec		dec	min	inc

b,c) Concavity:

		-1		0		1	
$12x$	−		−		+		+
$x-1$	−		−		−		+
$x+1$	−		+		+		+
$g''(x)$	−		+		−		+
$g(x)$	CD		CU		CD		CU

[see graph on back]

$\Rightarrow x = -1, 1$ are inflection pts.

Second Derivative Test:

$x = -\sqrt{2} \simeq -1.4$ CD $\frown \Rightarrow$ max, $x = +\sqrt{2} \simeq 1.4$ CU $\smile \Rightarrow$ min, $x = 0$ $g''(0) = 0$ $\underline{\underline{\text{inconclusive}}}$

This test fails for $x = 0$, but First derivative Test shows $x = 0$ is neither max nor min

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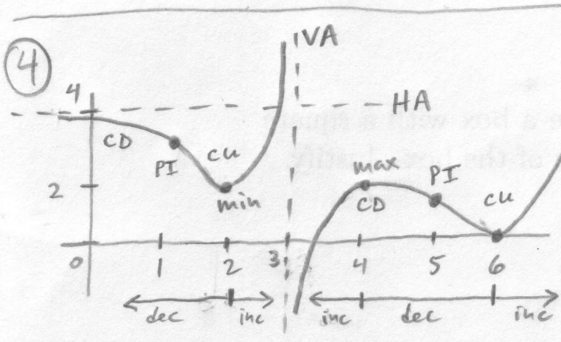
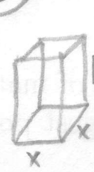
④



⑤ Area $= 12 = x^2 + 4xh \Rightarrow h = \frac{12-x^2}{4x}$

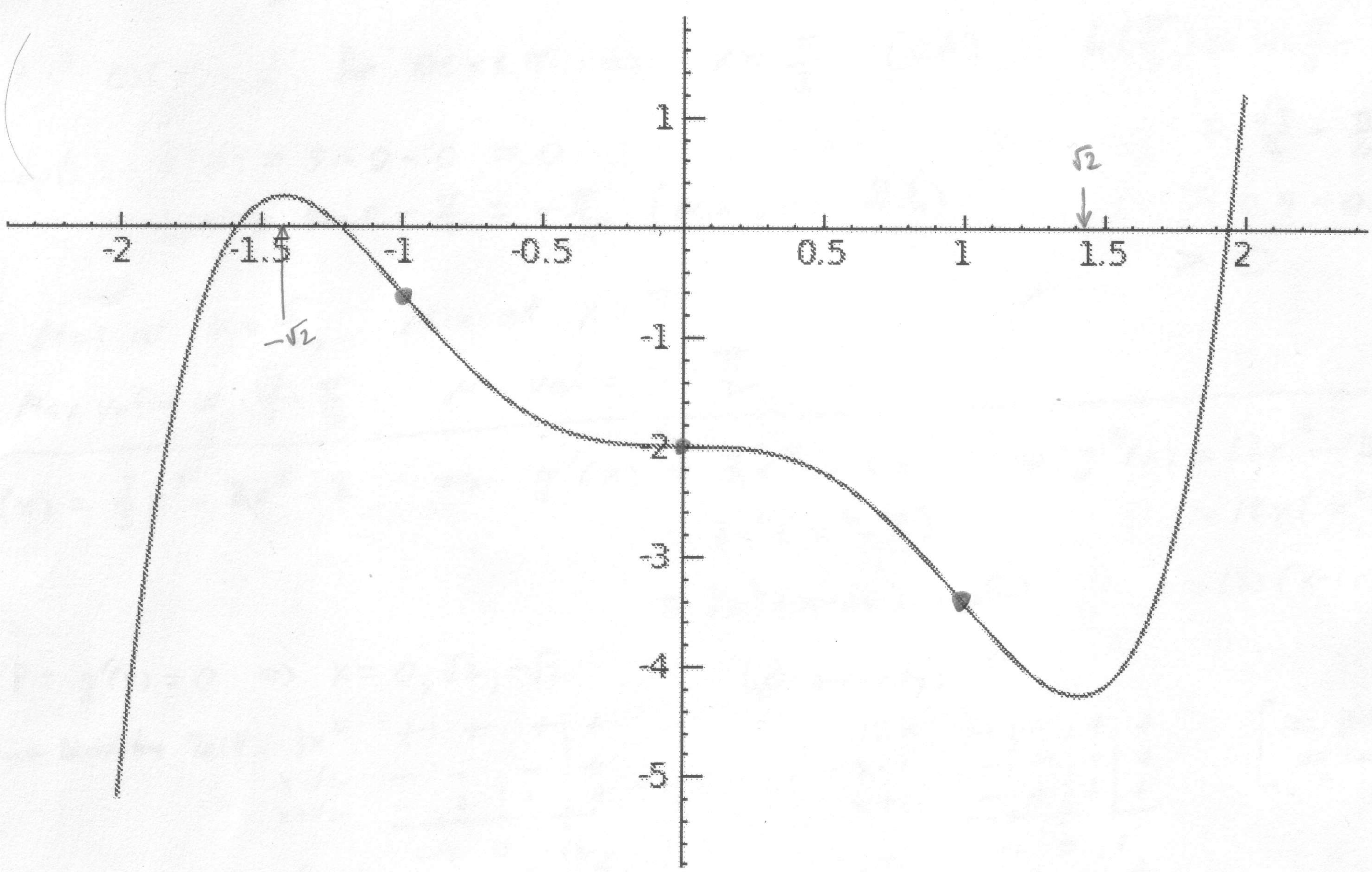


$V = x^2 h = x^2\left(\frac{12-x^2}{4x}\right) = 3x - \frac{x^3}{4}$

$V'(x) = 3 - \frac{3x^2}{4} \overset{\text{set}}{=} 0 \Rightarrow \frac{3x^2}{4} = 3 \Rightarrow \underline{\underline{x=2}}$

(Verify $V''(2) = -6(2)/4 < 0 \Rightarrow$ max)

$\Rightarrow h = \frac{12-4}{8} = 1$, so $V(2) = 2^2 \cdot 1 = \underline{4\,m^3}$



1
-1
-2
-3
-4
-5
-2
-1.5
-1
-0.5
0.5
1
1.5
2
$\sqrt{2}$
$-\sqrt{2}$