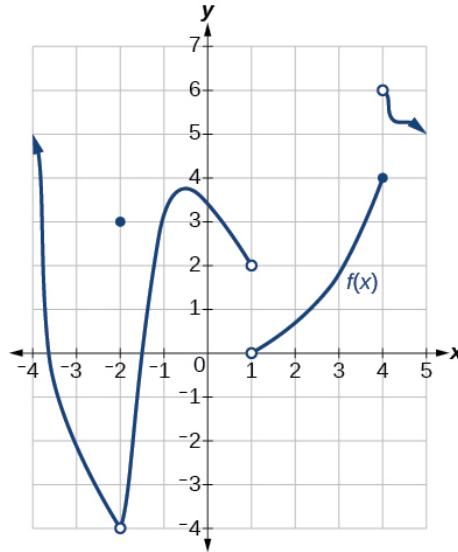# Business Calculus I (Math 221) Exam 1

March 4, 2015   Professor Ilya Kofman

Justify answers and show all work for full credit. No calculators permitted on this exam.

NAME: _____

Problem 1 (20pts). *The graph of* $y = f(x)$ *is shown above. Evaluate each limit, or write DNE if the limit does not exist. No justifications are necessary for this problem.*

(a) $\lim\limits_{x \to -2} f(x) =$

(b) $\lim\limits_{x \to 1^-} f(x) =$

(c) $\lim\limits_{x \to 1} f(x) =$

(d) $\lim\limits_{x \to -3} f(x) =$

(e) $\lim\limits_{x \to 4^+} f(x) =$

(f) $\lim\limits_{x \to 4^-} f(x) =$

(g) *For* $f(x)$ *to be continuous at* $x = -2$, *we must set* $f(-2) =$

(h) *Estimate the derivative* $f'(0) =$

(i) *Estimate the derivative* $f'(3.5) =$

(j) *Estimate for which* x *the derivative* $f'(x) = 0$, $x =$

Problem 2 (12pts). *Evaluate these limits. For an infinite limit, write $+\infty$ or $-\infty$.*
If a limit does not exist (DNE), you must justify. **Show all work!**

(a) $\displaystyle\lim_{x\to 6} \frac{x^2 - 2x - 24}{x^2 - 36}$

(b) $\displaystyle\lim_{x\to 1^-} \frac{1}{x+1}$

(c) $\displaystyle\lim_{x\to 1^-} \frac{1}{x-1}$

(d) $\displaystyle\lim_{x\to\infty} \frac{-8x^4 + 5x^2 - 2}{6x^4 + 3x^3 - 2x^2}$

Problem 3 (8pts). *Recall* $f'(a) = \displaystyle\lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$.
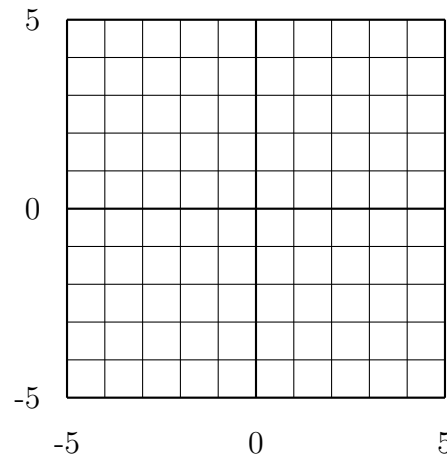
(a) *If* $f(x) = \sqrt{2x}$, *write the limit for* $f'(3)$. *Do not evaluate this limit.*

(b) *Show that* $g(x) = |x|$ *is not differentiable at* 0. *Evaluate this limit.* **Show all work!**

Problem 4 (5pts). **(a)** *On the grid below, graph the following piecewise defined function.*

$$f(x) = \begin{cases} 4 + 2x & x \leq -1 \\ 2 - x & x > -1 \end{cases}$$

(b) *Is the function $f(x)$ continuous at $x = -1$? (Do not justify.)*   **YES**   **NO**

Problem 5 (6pts). *For what value of c (if any) is the function $g(x)$ continuous at $x = 2$? Justify your answer.*

$$g(x) = \begin{cases} x^3 - \dfrac{2x - 1}{3} & x < 2 \\ c & x = 2 \\ x^2 + \dfrac{3x}{2} & x > 2 \end{cases}$$

Problem 6 (24pts). *Compute the derivative* $y' = \dfrac{dy}{dx}$. *Do not simplify. Show all work!*

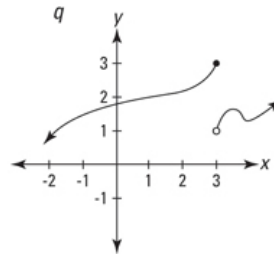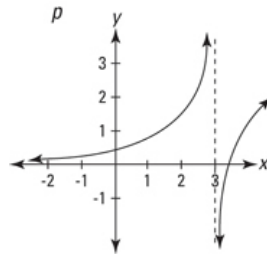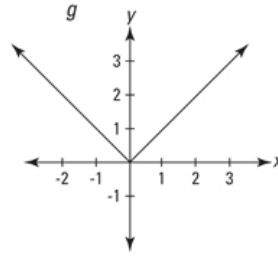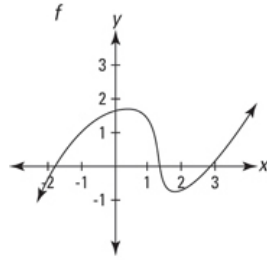(a) $y = \dfrac{x^3}{2} + 9x^{2/3} - 2x + 6 + 10x^{-1/2}$

(b) $y = \dfrac{4}{\sqrt[3]{x}} - 3\sqrt{x^5} + \dfrac{10}{x} + \dfrac{5}{x^6}$

(c) $y = \sqrt{5x^3 - 4x^2 - 3}$

(d) $y = \dfrac{8x^4 + 7x^3}{x^6 - 3}$

(e) $y = (3x^4 + 2x^3 + 7)(5x^9 - 8)$

(f) $y = (6 + \sqrt[3]{x - 4})^{-4/5}$

Problem 7 (10pts). *Circle every label for which the statement for that graph is true.*

(a) *The graph is continuous for all x shown.*   **f**   **g**   **p**   **q**

(b) *The graph is differentiable for all x shown.*   **f**   **g**   **p**   **q**

(c) *For some x shown, the derivative is zero.*   **f**   **g**   **p**   **q**

(d) *For all x where the derivative exists, it is positive.*   **f**   **g**   **p**   **q**

(e) *The derivative of the graph at $x = 0$ is positive.*   **f**   **g**   **p**   **q**

Problem 8 (5pts). *Let $F(x) = 2x^3 - x^2 + 1$. Find the equation of the tangent line to the graph of $F(x)$ at $x = 2$. Leave your answer in the form $y = mx + b$.*

Problem 9 (8pts). *Let* $g(x) = (3x - 1)^4$.

(a) *Find* $g'(0)$.

(b) *Find* $g''(0)$.

Problem 10 (12pts). *For x units sold, the total revenue function is $R(x) = 42x + 200$. The total cost function is $C(x) = 1000 + 30x + \dfrac{1}{5}x^2$.*

(a) *Find the profit function $P(x)$.*

(b) *Find the marginal profit when 10 units are sold.*

(c) *If $P(10) = -700$, use your answer in part (b) to estimate the total profit if 11 units sold.*

(d) *Should the company sell the 11th unit? Explain using your answers above.*