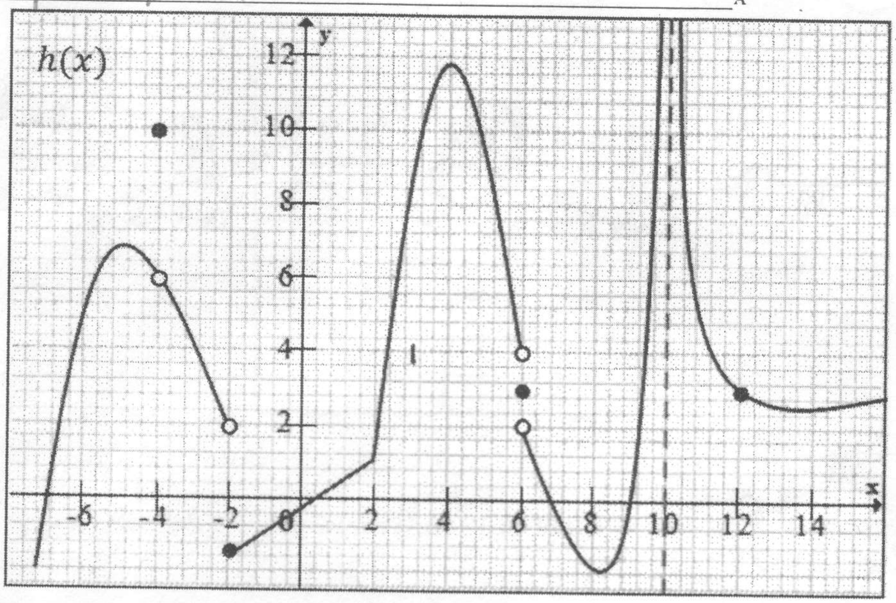
Business Calculus I (Math 221) Exam 1

September 28, 2016 — Professor Ilya Kofman

Justify answers and show all work for full credit. No calculators permitted on this exam.

NAME: Key



Problem 1 (20pts). *The graph of $y = f(x)$ is shown above. Evaluate each limit, or write DNE if the limit does not exist. No justifications are necessary for this problem.*

(a) $\lim_{x\to -4} f(x) =$ 6

(b) $\lim_{x\to 10^-} f(x) =$ $+\infty$

(c) $\lim_{x\to 2} f(x) =$ 1

(d) $\lim_{x\to -2} f(x) =$ DNE

(e) $\lim_{x\to 6^+} f(x) =$ 2

(f) $\lim_{x\to 6^-} f(x) =$ 4

(g) *For $f(x)$ to be continuous at $x = -4$, we must set* $f(-4) =$ 6

(h) *Estimate the derivative* $f'(0) =$ 1/2 or 1

(i) *Estimate the derivative* $f'(4) =$ 0

(j) *Estimate the derivative* $f'(12) =$ -1

2 pts each

110

20

Problem 2 (12pts). *Evaluate these limits. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify.* **Show all work!**

3pts. each

(a) $\lim_{x\to 3} \frac{x^2-11x+24}{x-3}$ $= \lim_{x\to 3} \frac{\cancel{(x-3)}(x-8)}{\cancel{x-3}} = \lim_{x\to 3} x-8 = -5$

(b) $\lim_{x\to -5} \frac{x^2+x-20}{x^2-25}$ $= \lim_{x\to -5} \frac{\cancel{(x+5)}(x-4)}{\cancel{(x+5)}(x-5)} = \lim_{x\to -5} \frac{x-4}{x-5} = \frac{-9}{-10} = \frac{9}{10}$

(c) $\lim_{x\to 4^-} \frac{1}{3x-12}$ $= -\infty$

$\to 0$

$(3x-12<0)$

(d) $\lim_{x\to\infty} \frac{-5x^6+2x^2-2}{4x^6+3x^3-2x}$ $= \lim_{x\to\infty} \frac{-5+2/x^4-2/x^6}{4+3/x^3-2/x^5} = -\frac{5}{4}$

Problem 3 (8pts). *Recall* $f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$.

3

(a) *If $f(x) = 5x^3$, write the limit for $f'(2)$. Do not evaluate this limit.*

$\lim_{h\to 0} \frac{5(2+h)^3 - 5(2)^3}{h} = \lim_{h\to 0} \frac{5(2+h)^3 - 40}{h}$

5

(b) *Show that $g(x) = |x|$ is not differentiable at 0. Evaluate this limit.* **Show all work!**

$g'(0) = \lim_{h\to 0} \frac{|0+h|-|0|}{h} = \lim_{h\to 0} \frac{|h|}{h}$

$\lim_{h\to 0^+} \frac{|h|}{h} = \lim_{h\to 0^+} \frac{h}{h} = 1$

$\lim_{h\to 0^-} \frac{|h|}{h} = \lim_{h\to 0^-} -\frac{h}{h} = -1$

$\Rightarrow g'(0) = \lim_{h\to 0} \frac{|h|}{h}$ DNE

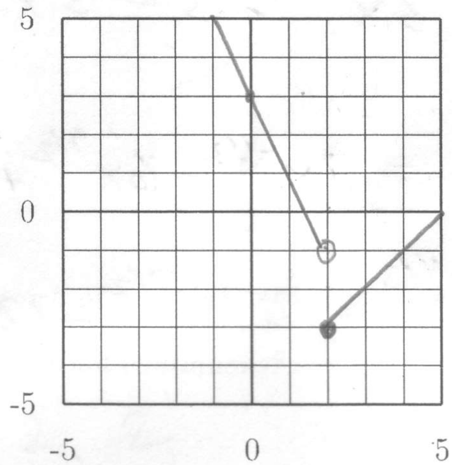
20

Problem 4 (5pts). **(a)** *On the grid below, graph the following piecewise defined function.*

$$f(x) = \begin{cases} 3-2x & x<2 \\ x-5 & x \geq 2 \end{cases}$$

(b) *Is the function $f(x)$ continuous at $x=2$? (Do not justify.)* **YES** NO

1



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Problem 5 (6pts). *For what value of c (if any) is the function $g(x)$ continuous at $x=3$? Justify your answer.*

$$g(x) = \begin{cases} \dfrac{8x-4}{x^2+1} & x<3 \\ c & x=3 \\ x^2-4x+5 & x>3 \end{cases}$$

$$\lim_{x\to 3^-} g(x) = \lim_{x\to 3^-} \frac{8x-4}{x^2+1} = \frac{20}{10} = 2$$

4

$$\lim_{x\to 3^+} g(x) = \lim_{x\to 3^+} x^2-4x+5 = 9-12+5 = 2$$

$$\Rightarrow \lim_{x\to 3} g(x) = 2$$

So let $g(3) = c = 2$

2

Then $\lim_{x\to 3} g(x) = g(3)$.

11

Problem 6 (24pts). *Compute the derivative* $y' = \frac{dy}{dx}$. *Do not simplify. Show all work!*

(a) $y = \frac{x^5}{3} - 4x^{3/4} + 3x + 8 + 15x^{-1/3}$

$$y' = \frac{5}{3}x^4 - 3x^{-1/4} + 3 - 5x^{-4/3}$$

(b) $y = \frac{7}{\sqrt[3]{x}} - 6\sqrt{x^9} + \frac{12}{x} + \frac{4}{x^7}$ $= 7x^{-1/3} - 6x^{9/2} + 12x^{-1} + 4x^{-7}$

$$y' = -\frac{7}{3}x^{-4/3} - 27x^{7/2} - 12x^{-2} - 28x^{-8}$$

(c) $y = \sqrt[3]{2x^5 - 3x^2 - 2}$

$$y' = \frac{1}{3}(2x^5 - 3x^2 - 2)^{-2/3}(10x^4 - 6x)$$

(d) $y = \frac{6x^4 + 5x^3}{x^6 - 2}$

$$y' = \frac{(x^6-2)(24x^3+15x^2) - (6x^4+5x^3)(6x^5)}{(x^6-2)^2}$$

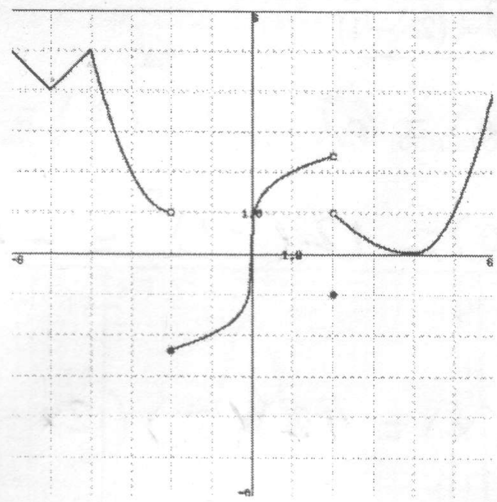
(e) $y = (3x^5 + 4x^4 - 2)(4x^7 - 18)$

$$y' = (3x^5+4x^4-2)(28x^6) + (15x^4+16x^3)(4x^7-18)$$

(f) $y = \sqrt{(3x + 2)^4 - 15x}$

$$y' = \frac{1}{2}((3x+2)^4 - 15x)^{-1/2}(\underbrace{4(3x+2)^3(3)}_{12(3x+2)^3} - 15)$$

24



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Problem 7 (~~10~~pts). *The graph of $y = f(x)$ is shown above for $-6 < x < 6$.*

(a) *For which x values is $f(x)$ not continuous?*

$x = -2, 2$ 2

(b) *For which x values is $f(x)$ not differentiable?*

$x = -5, -4, -2, 0, 2$ 4

(c) *For which x values is the derivative $f'(x) = 0$?*

$x = 4$ 2

Problem 8 (~~7~~pts). *Let $F(x) = 3x^3 - 2x^2 - 10$. Find the equation of the tangent line to the graph of $F(x)$ at $x = 1$. Leave your answer in the form $y = mx + b$.*

$m = F'(1) = (9x^2 - 4x)\big|_{x=1} = 9 - 4 = 5$ 3

$F(1) = 3 - 2 - 10 = -9 \quad (1, -9)$ 2

$y + 9 = 5(x - 1)$

$\underline{y = 5x - 14}$ 2

15

Problem 9 (8pts). *Let* $g(x) = (2x-1)^6$.

(a) *Find* $g'(0)$.

$g'(x) = 6(2x-1)^5(2) = 12(2x-1)^5$

$g'(0) = 12(-1)^5 = -12$

4

(b) *Find* $g''(0)$.

$g''(x) = (12)(5)(2x-1)^4(2)$

$g''(0) = (12)(5)(-1)^4(2) = 120$

4

Problem 10 (12pts). *For* x *units sold, the total revenue function is* $R(x) = 30x + 100$. *The total cost function is* $C(x) = 500 + 8x + \frac{1}{8}x^2$.

(a) *Find the profit function* $P(x)$.

$P(x) = R(x) - C(x) = (30x+100) - (500 + 8x + \frac{1}{8}x^2)$

(b) *Find the marginal profit when 100 units are sold.* $P(x) = -\frac{1}{8}x^2 + 22x - 400$

1

$P'(x) = -\frac{1}{4}x + 22$

$P'(100) = -\frac{1}{4}(100) + 22 = -25 + 22 = -3$

4

(c) *If* $P(100) = 550$, *use your part (b) answer to estimate the total profit if 101 units sold.*

$P(101) \approx P(100) + P'(100) = 550 - 3 = 547$

3

(d) *Should the company sell the 101st unit? Explain using your answers above.*

NO, $P'(100) < 0$ Marginal profit is negative so profit will decrease from sale of 101^{st} unit.

4

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