

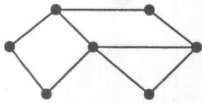
Mathematics for Liberal Arts (Math 102) Exam 1

Date: February 22, 2007

Professor Ilya Kofman

NAME: Key

Problem 1. Answer the questions that are below each graph:



(a) This graph has an Euler circuit.

~~T~~  F

(b) This graph has an Euler path, which is not a circuit.

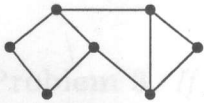
T  F

(c) How many edges must be added to best Eulerize this graph?

2

(d) How many edges will a spanning tree for this graph have?

6



(e) This graph has an Euler circuit.

T  F

(f) This graph has an Euler path, which is not a circuit.

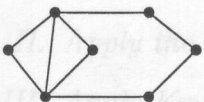
T  F

(g) How many edges must be added to best Eulerize this graph?

2

(h) How many edges will a spanning tree for this graph have?

6



(i) This graph has an Euler circuit.

T  F

(j) This graph has an Euler path, which is not a circuit.

T  F

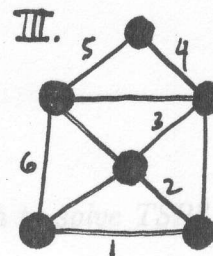
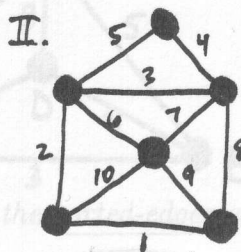
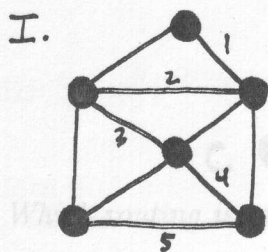
(k) How many edges must be added to best Eulerize this graph?

0

(l) How many edges will a spanning tree for this graph have?

6

**Problem 2.** Consider the paths given by the sequences of numbered edges on the graphs as shown. Circle the number of every graph whose path has the property:



- |  |   |    |     |      |
|--|---|----|-----|------|
| (a) The path is a circuit.                   | I | II | III | None |
| (b) The path is a spanning tree.             | I | II | III | None |
| (c) The path covers every edge exactly once. | I | II | III | None |
| (d) The path is an Euler circuit.            | I | II | III | None |
| (e) The path is a Hamiltonian circuit.       | I | II | III | None |

**Problem 3.** If a connected graph has 16 vertices of odd valence, at least how many edges must be added to Eulerize the graph? 8

**Problem 4.** Does the complete graph  $K_{15}$  have an Euler circuit? Why or why not?

yes, all valences = 14 (even)

**Problem 5.** Which one of the following techniques should be applied in each case:

I. Find an Euler circuit or best Eulerization

II. Apply the sorted-edges algorithm to solve TSP

III. Apply Kruskal's algorithm to find minimal-cost spanning tree

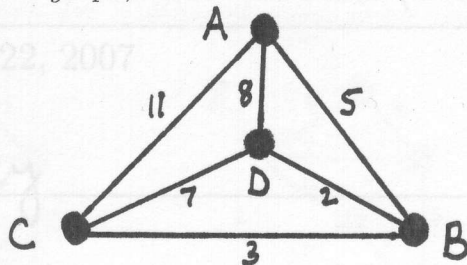
- |   |   |    |     |      |
|---|---|----|-----|------|
| (a) NYC subway fan wants to quickly visit every station.                              | I | II | III | None |
| (b) NYC subway fan wants to quickly see all graffiti on subway tunnel walls.          | I | II | III | None |
| (c) NYC wants to install new expensive fiber optic cable between all subway stations. | I | II | III | None |

Problem 6. For this graph, circle the correct answer below each question.

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(b) Which routing is produced by the sorted-edges algorithm to solve TSP?

$2+3+8+11 = 24$

- 1) ABCDA    2) ABDCA    3) ACBDA    4) ACDAB    5) ABDAC

(a) Which routing is produced by the nearest-neighbor algorithm to solve TSP?

$2+5+7+11 = 25$

- 1) ABCDA    2) ABDCA    3) ACBDA    4) ACDAB    5) ABDAC

(c) Which routing is produced by the brute-force algorithm to solve TSP?

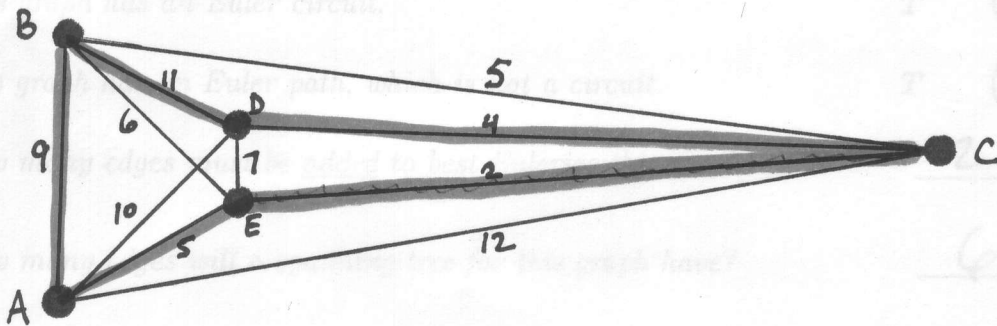
$3+5+7+8 = 23$

- 1) ABCDA    2) ABDCA    3) ACBDA    4) ACDAB    5) ABDAC

(d) Using Kruskal's algorithm, what is the cost of the spanning tree? 10

6pts  
4pts

Problem 7. Answer the questions for this graph. Show your work for full credit.



(a) What is the cost of the Hamiltonian circuit obtained by using the sorted-edges algorithm? \_\_\_\_\_

$2+4+5+9+11 = 31$

5pts.  
computed wrong circuit 2/5

(b) What is the cost of the minimal-cost spanning tree using Kruskal's algorithm? \_\_\_\_\_

$2+4+5+5 = 16$

5pts.  
Computed spanning tree 3/5