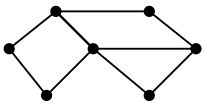
Mathematics for Liberal Arts (Math 102) Exam 1

Date: February 22, 2007 Professor Ilya Kofman

NAME: _____

Problem 1. *Answer the questions that are below each graph:*

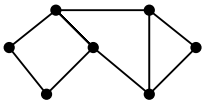


(a) *This graph has an Euler circuit.* ***T*** ***F***

(b) *This graph has an Euler path, which is not a circuit.* ***T*** ***F***

(c) *How many edges must be <u>added</u> to best Eulerize this graph?* _____

(d) *How many edges will a spanning tree for this graph have?* _____

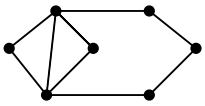


(e) *This graph has an Euler circuit.* ***T*** ***F***

(f) *This graph has an Euler path, which is not a circuit.* ***T*** ***F***

(g) *How many edges must be <u>added</u> to best Eulerize this graph?* _____

(h) *How many edges will a spanning tree for this graph have?* _____



(i) *This graph has an Euler circuit.* ***T*** ***F***

(j) *This graph has an Euler path, which is not a circuit.* ***T*** ***F***

(k) *How many edges must be <u>added</u> to best Eulerize this graph?* _____

(l) *How many edges will a spanning tree for this graph have?* _____

Problem 2. *Consider the paths given by the sequences of numbered edges on the graphs as shown. Circle the number of every graph whose path has the property:*

(a) *The path is a circuit.*	***I***	***II***	***III***	***None***
(b) *The path is a spanning tree.*	***I***	***II***	***III***	***None***
(c) *The path covers every edge exactly once.*	***I***	***II***	***III***	***None***
(d) *The path is an Euler circuit.*	***I***	***II***	***III***	***None***
(e) *The path is a Hamiltonian circuit.*	***I***	***II***	***III***	***None***

Problem 3. *If a connected graph has* 16 *vertices of odd valence, at least how many edges must be added to Eulerize the graph?* _____

Problem 4. *Does the complete graph* K_{15} *have an Euler circuit? Why or why not?*

Problem 5. *Which one of the following techniques should be applied in each case:*

I. Find an Euler circuit or best Eulerization

II. Apply the sorted-edges algorithm to solve TSP

III. Apply Kruskal's algorithm to find minimal-cost spanning tree

(a) *NYC subway fan wants to quickly visit every station.*	***I***	***II***	***III***	***None***
(b) *NYC subway fan wants to quickly see all graffiti on subway tunnel walls.*	***I***	***II***	***III***	***None***
(c) *NYC wants to install new expensive fiber optic cable between all subway stations.*	***I***	***II***	***III***	***None***

Problem 6. *For this graph, circle the correct answer below each question.*

(a) *Which routing is produced by the nearest-neighbor algorithm to solve TSP?*

1) *ABCDA* 2) *ABDCA* 3) *ACBDA* 4) *ACDAB* 5) *ABDAC*

(b) *Which routing is produced by the sorted-edges algorithm to solve TSP?*

1) *ABCDA* 2) *ABDCA* 3) *ACBDA* 4) *ACDAB* 5) *ABDAC*

(c) *Which routing is produced by the brute-force algorithm to solve TSP?*

1) *ABCDA* 2) *ABDCA* 3) *ACBDA* 4) *ACDAB* 5) *ABDAC*

(d) *Using Kruskal's algorithm, what is the cost of the spanning tree?* _____

Problem 7. *Answer the questions for this graph. Show your work for full credit.*

(a) *What is the cost of the Hamiltonian circuit obtained by using the sorted-edges algorithm?* _____

(b) *What is the cost of the minimal-cost spanning tree using Kruskal's algorithm?* _____