Calculus IIIS/IVA, Midterm 2. Your name: ____________________

April 4, 2001, 10:35-11:50am, 703 Hamilton Hall Your ID#: ____________________

This exam consists of 6 problems. Calculators are not allowed in the exam. Show your work to receive credits. You can ask the proctor for extra scratch papers.

1. (15pts) Evaluate the double integral $\iint_R x^2 dA$, where R is the region bounded by the lines $x + 2y = 1, x + 2y = 4, -2x + y = 0$, and $-2x + y = 2$.

2. Let $\mathbf{F}(x, y, z) = (2x + y^2 \sin z, 2yz + 2xy \sin z, y^2 + xy^2 \cos z)$ be a vector field and $\mathbf{r}(t) = (t^2, t^2 + t - 2, t^2 - t),\ 0 \leq t \leq 1$ be a curve.
   (a) (5pts) Is $\mathbf{F} = \operatorname{curl} \mathbf{G}$ for some vector field $\mathbf{G}$? Justify your answer.
   (b) (10pts) Find a function such that $\mathbf{F} = \nabla f$.
   (c) (10pts) Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$.

3. (15pts) Evaluate the surface integral $\iint_S (x^2 + y^2) dS$, where S is the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies above the xy-plane.

4. (15pts) Evaluate the surface integral $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = (x^4 + z^2, x + \sin(y^2), z^2 + zx^3)$ and S is the upper hemi-sphere $x^2 + y^2 + z^2 = 4, z \geq 0$ with outward orientation.

5. (15pts) Use the Divergence Theorem to evaluate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = (xy, xy, 3y + x^3)$ and S is the surface of the solid inside the cylinder $x^2 + y^2 = 4$ and bounded by $z = 0$ and $z = 1 + x^2 + y^2$.

6. (15pts) Evaluate the line integral

$$\int_C \frac{-y dx}{x^2 + y^2} + \frac{x dy}{x^2 + y^2},$$

   where C is the boundary (counterclockwise orientation) of the square with vertices $(-1, -1), (1, -1), (1, 1)$, and $(-1, 1)$.