

/ 60 pts

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Mathematics V1205x, Calculus IIIS/IVA

Midterm Examination #2: March 28, 2001, 9:10–10:35 am

Name: _____

The following midterm has 4 problems which are each worth 15 points. Please read the exam carefully and check all your answers. Show all your work and justify your steps.

1. Suppose $F = 2xyi + (x^2 + byz)j + y^2k$.

A. For what number b does $\text{div}(F) = \text{curl}(F) \cdot \mathbf{i}$?

B. For what (different) number b is F conservative?

C. For this second number b , find f so that $\nabla(f) = F$.

2. A. Rewrite the following integral as an iterated integral in the order $dydx dz$:

$$\int_{-1}^1 \int_{x^2}^1 \int_0^{1-y} f(x, y, z) dz dy dx.$$

- B. Rewrite the integral in spherical coordinates. DO NOT INTEGRATE.

$$\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} z dz dy dx.$$

- C. Evaluate $\iint_R e^{9x^2+4y^2} dA$ where R is the region bounded by $9x^2 + 4y^2 = 1$.

3. A. (10 points) Let $F = \langle 1 + \tan(x), x^2 + e^y \rangle$ be a force field. Let C be the boundary of the region enclosed by the parabola $x = y^2$ and the lines $x = 1$ and $y = 0$. Find the work done by F as a particle travels once around C in the counterclockwise direction.

- B. (5 points) Find a vector field F such that $\int_C F \cdot dr = 0$ whenever the endpoints of C both lie on the curve $y = x^3 + x + 1$.

4. Consider the surface S in \mathbb{R}^3 given parametrically by $x = u \cos(v)$, $y = u \sin(v)$, and $z = u$. Let (u, v) range through the domain $D = \{(u, v) | 0 \leq u \leq 1, 0 \leq v \leq 2\pi\}$.

A. Graph S . Mark the grid curves $u = 1$ and $v = 0$.

B. Find the surface area (for $(u, v) \in D$).

C. Let C be the grid curve $v = 0$, $0 \leq u \leq 1$. Find $\int_C 1 ds$. What physical quantity does this integral represent?